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Analysis of Mode Mismatch in Quantum Scissors Device

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Abstract– Effect of mode mismatch between the input fields of a quantum scissors device is studied using the continuum mode representation. It has been observed that mode mismatch has a large deteriorating effect when a strong coherent state is used to prepare the desired state.

I. Introduction–Possible applications of nonclassical states in quantum communication and information processing has stimulated a growing interest in the generation and engineering of Fock states and their superpositions. Such states can be prepared using either nonlinear medium or conditional measurements at the output ports of an array of beamsplitters [1]. Quantum scissors device (QSD) proposed by Pegg *et al.* is a relatively simpler scheme for the preparation of superposition states in the form $a_0|0\rangle + a_1|1\rangle$ [2]. This brilliant scheme seen in Fig.1 is based on the generation of the desired superposition state by truncating a coherent input state. Conditional measurement at two photon counters placed at the output ports of a beam splitter after mixing the coherent state with one of the modes of an entangled photon pair at this beam splitter forms the basic principle behind this scheme. Recently, we have shown that the QSD scheme is practically realizable with the available technology of photon counting and single photon generation with the condition that the coherent state to be truncated has low intensity [3]. However, in that study, the analysis was performed considering that the coherent and single photon states are in perfectly matching single modes. Since the scheme is based on the interference of lights from independent sources (e.g., single photon from parametric down conversion and a coherent light), the two input modes can be different and a good mode matching may constitute a major challenge in experiment. A theoretical study to understand the effects of mismatch in QSD scheme is necessary. In this paper, we present the results of this study and discuss mode match effects on the fidelity of truncation.

II. Continuum States of the QSD–For the study of mode mismatching, it is necessary to use a multimode theory. Here we use the continuum mode formalism developed in [4]. In QSD scheme shown in Fig.1, we assume that the single photon state at the input mode of $BS1$ is prepared in a mode defined with a mode function of $\xi(\omega)$ and the coherent state at $BS2$ is prepared in $\beta(\omega)$. Using the general formalism introduced in [4], the input states of the QSD can be written as

$$|1;\xi\rangle_{a_1} = \hat{A_1}^{\dagger}(\xi)|0\rangle_{a_1} \qquad (1)$$

$$|\alpha;\beta\rangle_{b_3} = e^{-|\alpha|^2/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{n!}[\hat{B_3}^{\dagger}(\beta)]^n|0\rangle_{a_3} \qquad (2)$$

for the single photon state and the coherent state, respectively. $\hat{A_1}^{\dagger}$ and $\hat{B_3}^{\dagger}$ are called "field operators" and given as

$$\hat{A_1}^{\dagger}(\xi) = \int d\omega\xi(\omega)\hat{a_1}^{\dagger}(\omega) \qquad (3)$$

$$\hat{B_3}^{\dagger}(\beta) = \int d\omega\beta(\omega)\hat{b_3}^{\dagger}(\omega) \qquad (4)$$

where $\hat{a_1}^{\dagger}$ and $\hat{b_3}^{\dagger}$ are the creation operators of the corresponding input modes of $BS1$. The commutator between field operators of different modes corresponds to the overlap between the mode of single photon state and that of the coherent state. Using Gram- Schmidt othogonalization method, we can decompose ξ as $\xi = \gamma_0\beta + \gamma_1\sigma$ where $\gamma_0 = (\xi,\beta)$ and $\gamma_1 = (\xi,\sigma)$. σ is orthogonal to β and can be calculated using

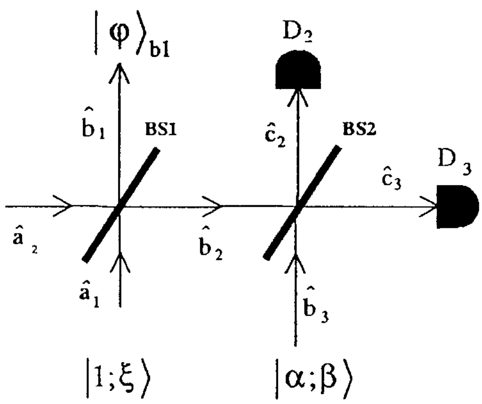

Figure 1: Basic configuration of QSD. The states at BS inputs are the continuum single photon and coherent states with modes ξ and β, respectively.

$$\sigma = \frac{\xi - \gamma_0\beta}{\sqrt{1-|\gamma_0|^2}} \tag{5}$$

Then any field operator of mode ξ can be decomposed in terms of modes σ and β as follows $\hat{B}^\dagger(\xi) = \gamma_0\hat{B}^\dagger(\beta) + \gamma_1\hat{B}^\dagger(\sigma)$. With the input state $|1;\xi\rangle_{a_1}|0\rangle_{a_2}$ at $BS1$, the resultant output state at the output of $BS1$ can be written as

$$\begin{aligned}|\psi\rangle_{b_1b_2} = {} & t_1[\gamma_0|1;\beta\rangle_{b_1}|0\rangle_{b_2} + \gamma_1|1;\sigma\rangle_{b_1}|0\rangle_{b_2}] \\ & + r_1[\gamma_0|0\rangle_{b_2}|1;\beta\rangle_{b_1} + \gamma_1|0\rangle_{b_2}|1;\sigma\rangle_{b_1}]\end{aligned} \tag{6}$$

Using the input output relation for $BS2$ together with Eq. (1)-(4) and performing a conditional measurement with the photon counters at the outputs, one can find the truncated output state. From the previous works, it is known that for 50:50 beamsplitters and ideal photon counters, a single photon detection at $D3$ and no photon detection at $D2$ will give the desired truncated state with fidelity equals to 1 provided that perfect mode matching is achieved. However, here, due to the mismatch introduced by σ mode, there will be a distortion in the truncated output state. We consider two cases; (i) both detectors are perfectly mode matched to β mode, and (ii) detectors have broad mode detection and can detect photons in both β and σ modes. Upon detection of one photon at detector $D3$ and 0 photon at detector $D2$, the unnormalized output truncated states for these two cases will be

$$\begin{aligned}|\varphi_1\rangle_{b_1} = {} & r_1[\gamma_0 t_2 + \alpha\gamma_0 r_2(t_2+r_2)]|0\rangle_{b_1} + \\ & \alpha r_2 t_1[\gamma_0|1;\beta\rangle_{b_1} + \gamma_1|1;\sigma\rangle_{b_1}]\end{aligned} \tag{7}$$

$$\begin{aligned}|\varphi_2\rangle_{b_1} = {} & r_1 t_2[\gamma_0 + \gamma_1]|0\rangle_{b_1} + \\ & \alpha r_2 t_1[\gamma_0|1;\beta\rangle_{b_1} + \gamma_1|1;\sigma\rangle_{b_1}]\end{aligned} \tag{8}$$

The single photon state in mode σ at the output states must be noted as the effect of mode mismatch. The fidelity of the generated output state to the desired one

$$|\Phi\rangle_{b_1} = \frac{1}{\sqrt{1+|\alpha|^2}}[|0\rangle_{b_1} + \alpha|1;\beta\rangle_{b_1}] \tag{9}$$

is calculated using $F_{i=1,2} = ||\langle\Phi|\varphi_{i=1,2}\rangle_{b_1}||^2$ for both cases. The plots of these calculations are given in Fig.2 and Fig.3 as a function of mismatch parameter $|\gamma_1|$ for various input coherent state intensities $|\alpha|^2$ (A)0.1, (B)0.5, (C)1, (D)2, (E)16. It can be seen from these figures that increasing intensities of coherent input state cause stronger degradation in fidelity of truncation for both cases. For the second case, with a weak intensity coherent state, mode mismatch causes only a slight decrease in fidelity of truncation; for a mismatch value of 0.8, fidelity is still higher than 0.9.

III. Conclusion–Mode mismatch effects in a QSD is studied using the continuum mode description of the input states. It has been understood that for detectors with unit efficiency and zero dark count, mode mismatch does not constitute a serious problem in the truncation of weak intensity coherent states.

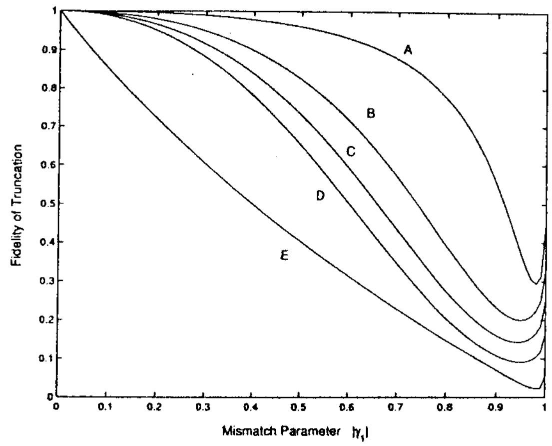

Figure 2: Fidelity of truncation with detectors perfectly mode matched to mode β.

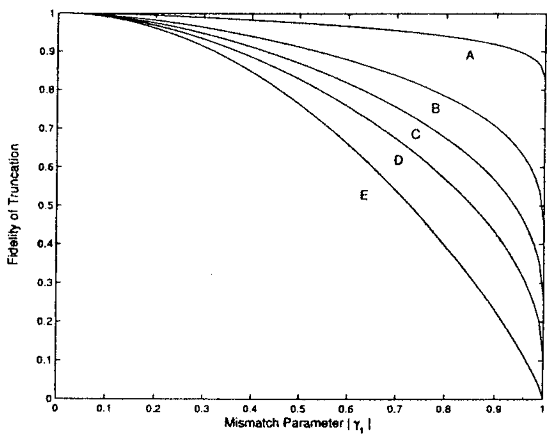

Figure 3: Fidelity of truncation with detectors which can count photons in both modes ξ and β.

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