Comparative study of resources for state preparation by projection synthesis

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Optical state truncation and generation using the projection synthesis method has been comparatively analyzed for the cases of ideal single photon source, single photon from parametric down conversion and a fainted coherent state as the resource. The comparison is done using the fidelity of the truncated state to the desired one for each of these resources.

I. INTRODUCTION

With the advent of quantum communication [1], teleportation [2] and quantum computation [3], the preparation and manipulation of nonclasical states of light have become increasingly important due to possible use of such states as qubits. The recently proposed method of projection synthesis, which is based on measuring particular properties of the light to engineer superposition states, has led to the practical scheme of *quantum scissors device* (QSD) [4,5]. This device has been shown to be capable of not only preparing superposition states of the form $c_0|0\rangle + c_1|1\rangle$ but for teleporting them, as well [6]. We have demonstrated that the scheme is realizable with the current level of experimental quantum optics [7]. The device prepares the state mentioned above by physical truncation of the photon number superposition making a coherent state.

The proposed scheme of QSD, seen in Fig. 1, needs two photon counting detectors and two 50:50 beam splitters (BS). A single photon state is impinged on the the first BS to form an entangled state of vacuum and one-photon states which will be used as the resource to truncate the coherent state. The entangled state and the coherent state are made to interfere at the second BS and photon counting is performed at the output ports. Detection of a photon by one of the detectors and none by the other will synthesize the state at the other output port of the first BS in the desired superposition state.

In this ideal configuration, the QSD scheme needs a single photon state which is a nonclassical state to synthesize another nonclassical state, namely the superposition of vacuum and one-photon states. Ideal single photon sources which generate exactly one photon for every cycle of the experiment are beyond the current technology of quantum device fabrication. In practice, there are two approaches to replace the ideal source: attenuated coherent light and conditioned single photon by parametric down conversion process which only approximate the ideal source. The effects of these approximations on the projection synthesis technique must be evaluated if they are to be used in experiments.

In this study, we present a comparative study of the effects of above mentioned single photon sources on the state truncation and preparation by projection synthesis. In section II, we will briefly review the QSD scheme and then in section III we will discuss the effects of the resources on the device. Conclusion will include a discussion of this study.

II. IDEAL STATE TRUNCATION PROCESS

Figure 1 depicts a schematic configuration of the original QSD proposal to generate a truncated state $|\Phi_{trunc}\rangle_{b_1} = \mathcal{N}(|0\rangle_{b_1} + \alpha|1\rangle_{b_1})$. The input state to the device is $|\Psi_{in}\rangle_{(a_1,a_2,b_3)} = |0\rangle_{a_1}|1\rangle_{a_2}|\alpha\rangle_{b_3}$ generating an intermediate state

$$|\psi\rangle_{(b_1,b_2)} = \frac{1}{\sqrt{2}}\left(|1\rangle_{b_1}|0\rangle_{b_2} + \mathrm{i}|0\rangle_{b_1}|1\rangle_{b_2}\right). \tag{1}$$

After the action of BS1 described by

$$\begin{pmatrix} \hat{b}_1^\dagger \\ \hat{b}_2^\dagger \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -\mathrm{i} \\ -\mathrm{i} & 1 \end{pmatrix} \begin{pmatrix} \hat{a}_1^\dagger \\ \hat{a}_2^\dagger \end{pmatrix} \tag{2}$$

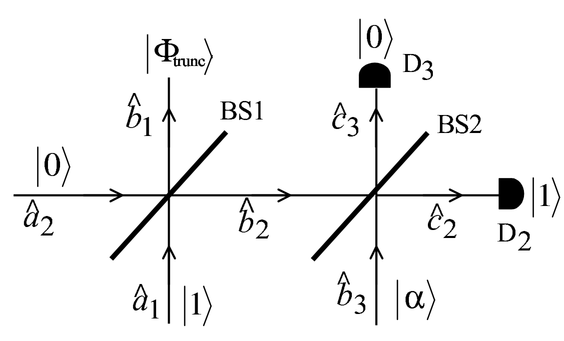

FIG. 1. Ideal QSD scheme using a perfect single photon source $|1\rangle$, ideal photon number resolving detectors D_2 and D_3, and the coherent state $|\alpha\rangle$. BS1 and BS2 are 50:50 symmetric beam splitters.

with $r = (i/\sqrt{2})$ and $t = (1/\sqrt{2})$. Then the three modes after the first step of unitary evolution can be written as

$$|\Psi\rangle = |\psi\rangle_{(b_1,b_2)} \otimes |\alpha\rangle_{b_3} = \mathrm{e}^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{2.n!}} \left[|10n\rangle_{(b_1 b_2 b_3)} + \mathrm{i}|01n\rangle_{b_1 b_2 b_3}\right]. \quad (3)$$

After the action of the BS2 whose action can be described by replacing $\hat{a}_1^\dagger$ $(\hat{a}_2^\dagger)$ by $\hat{b}_3^\dagger$ $(\hat{b}_2^\dagger)$ and $\hat{b}_1^\dagger$ $(\hat{b}_2^\dagger)$ by $\hat{c}_3^\dagger$ $(\hat{c}_2^\dagger)$ in Eq.(2), the output state is projected onto the one-photon detection by D_2 placed at mode $\hat{c}_2$ and no photon detection by D_3 placed at mode $\hat{c}_3$ of the BS2 resulting in the truncated output state of

$$|\Phi_{\mathrm{trunc}}\rangle_{b_1} = \frac{|0\rangle_{b1} + \alpha|1\rangle_{b1}}{\sqrt{1+|\alpha|^2}} \quad (4)$$

which is exactly the same as the superposition of the vacuum and one-photon components of the input coherent state. In the above calculations, D_2 and D_3 are assumed to be photon number resolving detectors with unit quantum efficiency. If the detectors are assumed to have efficiency η, then the output state will become [4]

$$\rho = \frac{[1+(1-\eta)|\alpha|^2]|0\rangle\langle 0| + |\alpha|^2|1\rangle\langle 1| + \alpha|1\rangle\langle 0| + \alpha^*|0\rangle\langle 1|}{1+|\alpha|^2(2-\eta)} \quad (5)$$

with a detection probability $P_d = 1+|\alpha|^2(2-\eta)$. Fidelity of such a truncation process becomes a function of η and the intensity $|\alpha|^2$ of the coherent light to be truncated.

III. STATE TRUNCATION WITH NON-IDEAL SINGLE PHOTON SOURCE

Ideal single photon sources which can generate exactly one photon on demand is not available yet. More practical ways of generating one-photon states are the conditioned parametric down conversion and attenuated (fainted) coherent light. In the following, we will first introduce the states generated by these methods and discuss how well they approximate the ideal single photon source and then present the results of state truncation using these methods:

A. Attenuated coherent light

In the case of attenuated coherent light, the number of photons N per pulse follows a poisson distribution which can be written as

$$p(N) = e^{-\bar{n}} \frac{\bar{n}^N}{N!} \quad (6)$$

with $\bar{n}$ being the average number of photons per pulse. For an ideal single photon source $p(N=1)=1$. To effectively use this source instead of ideal single photon source $\bar{n}$ must be made small enough to ensure the probability of generating more than one photon is negligible. However, making $\bar{n}$ smaller will result in increased probability of having zero photons in the generated pulses. Therefore $\bar{n}$ must be carefully chosen to make maximum use of the method. In Figs.2 and 3, we have depicted fidelity of truncation process for various $\bar{n}$ and intensities $|\alpha|^2$ of the light to be truncated. In this calculations, detectors are assumed to be photon number resolving detectors with unit quantum efficiency. It is clearly seen that, when the purpose is to truncate a coherent state to prepare a su-

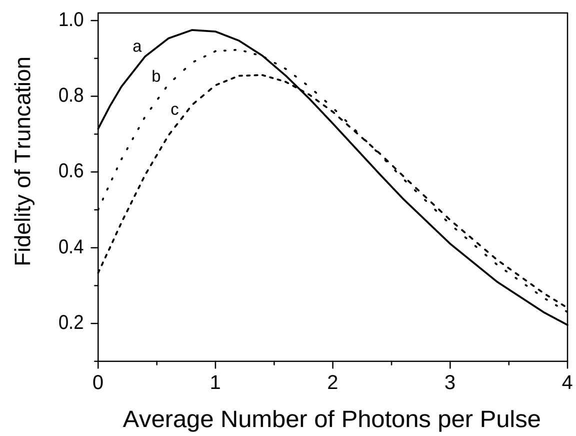

FIG. 2. Effect of average photon number per pulse $\bar{n}$ on the fidelity of state truncation when a fainted/attenuated coherent light is used to approximate the single photon source. Intensity of the input coherent states to be truncated are (a) $|\alpha|^2 = 0.4$, (b) $|\alpha|^2 = 1.0$ and (c) $|\alpha|^2 = 2$.

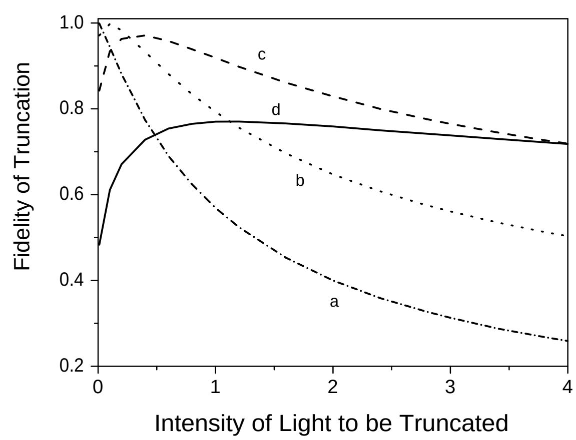

FIG. 3. Fidelity of truncating coherent input states of arbitrary intensity $|\alpha|^2$ using attenuated coherent state with various average photon number per pulse (a) $\bar{n} = 0.1$, (b) $\bar{n} = 0.5$, (c) $\bar{n} = 1$, and (d) $\bar{n} = 2.0$.

perposition state of the form $N(|0\rangle_{b_1} + \alpha|1\rangle_{b_1})$, for each $|\alpha|^2$ there can be found an $\bar{n}$ which will yield the maximum fidelity. When the light to be truncated has very weak intensity, then to achieve the highest attainable fidelity, one must use fainted coherent light of small $\bar{n}$ as small as $\bar{n} \sim 0.1$. However, it is understood that for $0.5 < |\alpha|^2 \leq 4$, $\bar{n} = 1$ performs the best. However, for much larger $|\alpha|^2$ values, $\bar{n}$ must also be increased.

The other strategy to use an attenuated coherent light to create a superposition state may be to avoid the QSD and then to attenuate the light to an optimized value which will give the highest fidelity to the desired state and use that light for the applications. The optimum value for $\bar{n}$ to obtain the maximum fidelity to the desired state of $[|0\rangle + \alpha|1\rangle]/\sqrt{1+|\alpha|^2}$ can be found as

$$\bar{n}_{\rm opt} = \frac{1 + 2|\alpha|^2 - \sqrt{1+4|\alpha|^2}}{2|\alpha|^2} \tag{7}$$

for which fidelity to the desired state becomes

$$F = \frac{e^{-\bar{n}_{\rm opt}}}{1+|\alpha|^2}(1 + 2|\alpha|\sqrt{\bar{n}_{\rm opt}} + \bar{n}_{\rm opt}|\alpha|^2). \tag{8}$$

B. Parametric down conversion

Parametric down conversion process, which is generally used for generating polarization, energy and momentum entangled photons, exploits the second order nonlinearities of materials of non-centrosymmetric. When an electromagnetic field is incident on such a material and undergoes the nonlinear interaction, the energy conservation condition will lead to the following output [8,9]

$$|\phi\rangle_{(i,s)} = \frac{1}{\cosh\kappa}\sum_{k=0}^{\infty}\tanh^k\kappa|k\rangle_i|k\rangle_s \tag{9}$$

where κ depends on the interaction time, strength of the nonlinearity and the pump energy. Eq.(9) clearly shows that if one photon is detected in mode i, then s mode should also contain one photon. In the limit of small κ, eq.(9) can be approximated to

$$|\phi\rangle_{(i,s)} \simeq \sqrt{1-\kappa^2}|0\rangle_i|0\rangle_s + \kappa|1\rangle_i|1\rangle_s \tag{10}$$

Averaging over all possible phases, the following mixed state is obtained for the output of parametric down conversion

$$\rho_{(i,s)} \simeq (1-\kappa^2)|00\rangle_{(i,s)(i,s)}\langle 00| + \kappa^2|11\rangle_{(i,s)(i,s)}\langle 11| \tag{11}$$

If κ is made to be small enough then contributions from higher order photon number states can be ignored. However, care must be taken because cost of this process is the decreased down conversion rate.

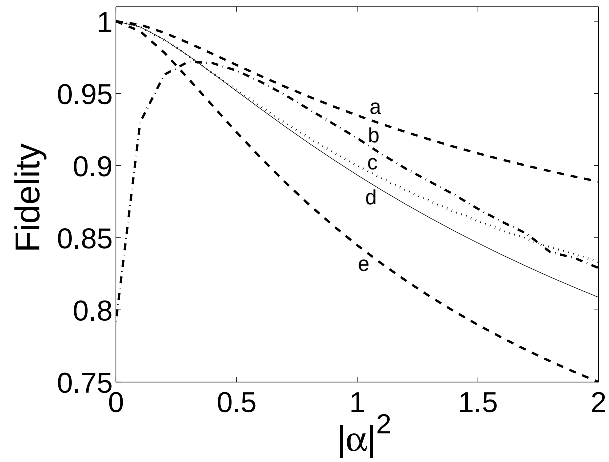

FIG. 4. Fidelity of state preparation using different strategies: parametric down conversion as the source (a) $\eta = 0.7$, (c) $\eta = 0.5$, (e) $\eta = 0.1$, (b) attenuated coherent light of $\bar{n} = 1.0$ to replace ideal photon source in QSD with detection efficiency of $\eta = 1$, and (d) optimized coherent state scheme without the use of QSD.

In order to use this as the input to the QSD device, another detector D_1 must be placed at the idler mode. In that case, to have a successful truncation process, we need to detect one photon states at detectors D_1 and D_2, and no photon detection at D_3. If we use photon number resolving counters with quantum efficiency η are used, then detection of one photon in the idler mode will leave the signal mode in the the

$$\rho_{(s)} \simeq \frac{{\rm Tr}_{\rm (i)}[\rho_{(i,s)}|1\rangle_{i\ i}\langle 1|]}{{\rm Tr}_{\rm (i,s)}[\rho_{(i,s)}|1\rangle_{i\ i}\langle 1|]} = |1\rangle_{s\ s}\langle 1| \tag{12}$$

with a detection probability of $P_1 = \eta\kappa^2$. Once this detection is achieved, the rest of the calculations follow similar to that given in section II. Then overall detection probability can be found by multiplying P_1 by that of P_d of section II resulting in $P_{pd} = \eta\kappa^2[1 + |\alpha|^2(2-\eta)]$. Proper adjustment of κ reduces the probability of generating more than one-photon pairs by parametric down conversion making the fidelity of truncation independent of κ. However, this reduces the probability of correct detection which will prepare the truncated state. When κ takes high values, more two-photon or more photon pairs can also be generated, resulting in a decreased fidelity which becomes a function of κ when detectors of $\eta \neq 1$ are used.

C. Comparison of effects of sources on fidelity of state preparation

In Figure 4, we have depicted the comparison of generating a state of the form Eq.(4) with different strategies.

It is seen that the maximum fidelity is obtained when the parametric down conversion is used with an ideal single photon source and detectors of $\eta \geq 0.7$ for all $|\alpha|^2$. For smaller η, fidelity decreases, however if $|\alpha|^2 \leq 0.3$ and $\eta \geq 0.5$, this scheme still performs better than all the other strategies. In this figure, we have depicted the attenuated coherent light with only $\bar{n} = 1.0$ which performs the worst of all the shown strategies for $|\alpha|^2 \leq 0.25$. With increasing $|\alpha|^2$, fidelity of this case increases and takes better values than all the other cases except the parametric down conversion source with detectors of $\eta \geq 0.7$. In a limited range of low $|\alpha|^2$, optimized light strategy gives better fidelity when low detector efficiency is used, but only with the cost of much lower fidelity outside the optimized range.

IV. CONCLUSIONS

In this study, we analyzed the state truncation by projection synthesis and compare different single photon sources suitable for this process using the fidelity. We studied attenuated coherent light and light from a parametric down conversion under special conditions to approximate the ideal single photon source. It is understood that parametric down conversion can approximate an ideal source provided that the κ parameter is chosen so that probability of producing pairs having more than one-photon in each mode is very close to zero. In this analysis, it is also seen that attenuated coherent light with an average photon number of 1 per pulse give fidelity values > 0.9 in a range of $0.1 \leq |\alpha|^2 \leq 1.3$. Analysis of these resources using quasi-distributions of the generated output states are left as future work.

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