Entanglement of excitonic states in coupled dots

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We discuss the entanglement of the excitonic states in the system of the coupled quantum dots (or artificial molecule) with fixed total exciton number by the entropy of entanglement. When the total exciton number is more than three, we cannot find maximally entangled state, but we find that the maximal values of the entropy of the entanglement for fixed exciton number is larger than that of beam splitting model with the same fixed number of photons when the initially prepared number of exciton is no more than ten.

I. INTRODUCTION

Recent developments in quantum computation and quantum information have created a large enthusiasm for theoretical and experimental physicists. Various theoretical and experimental researches have been conducted for preparation and measurement of the entangled states which have been considered to be a key ingredient in the realization of the quantum computer.

By quantum teleportation which was first realized by discrete variables [1] and then realized by continuous variables [2], we can quantitatively understand how quantum entanglement can be used as a resource for communication. The quantum teleportation with fixed total photon number as an entangled source has been theoretically investigated [3] and it has been understood that it is difficult to generate states with fixed photon number with today's technology. So the exploration of new entanglement sources with fixed particle number is very interesting and important both from experimental and theoretical points of views.

Motivated by these considerations, we study a system of two strong coupled quantum dots, which is also called artificial molecule [4,5]. In fact, several schemes which use coupled quantum dots have been proposed for fabricating quantum gate [6,7]. The quantum entanglement of the exciton states in a single quantum dot or in a quantum dot molecule is demonstrated experimentally [8,9]. The references [10,11] theoretically investigate the entanglement of excitonic states in the system of the optically driven coupled quantum dots and propose a method to prepare maximally entangled Bell and Greenberger-Horne-Zeilinger states.

In this paper we investigate the entanglement of the excitonic states in the artificial molecule with fixed exciton number in a closed quantum system. The paper is organized as follows: In Sec. II, we give a theoretical description of the artificial molecule. The analytical solution of the system is obtained by virtue of the Schwinger representation of the angular momentum. In Sec. III, the entanglement of the two subsystems is discussed using the von Neumann entropy under certain initial conditions. Finally, we give our conclusions and some comments.

II. THEORETICAL MODEL AND ANALYTICAL SOLUTION OF SYSTEM

We consider two completely symmetric semiconductor quantum dots which are coupled by Coulomb interaction. Then we apply two-band approximation to model these two coupled quantum dots. Within the two-band approximation, we have the Hamiltonian

$$H = \hbar \sum_{i,j=1}^{2} \chi_{ij} a_i^\dagger a_j + \sum_{i,j,k,l=1}^{2} \chi_{ijkl} a_i^\dagger a_j^\dagger a_k a_l, \qquad (1)$$

where $a_i^\dagger(a_i)$ are creation (annihilation) operators of excitons, which are electron-hole pairs bound by the Coulomb interaction. We assume that the density of the excitons is so low and the external confinement potential to quantum dot is so weak that exciton operators $a_i^\dagger(a_i)$ can be approximated by bosonic operators, that is, they satisfy the commutation relations of the ideal bosons, $[a_i, a_j^\dagger] = \delta_{ij}$, which is somewhat different from reference [12]. The label $i = 1(2)$ denotes the quantum dot $A(B)$. In the Hamiltonian (1), the deviation of the exciton operators from the ideal bosonic model are corrected by introducing an effective nonlinear interaction between the hypothetical ideal bosons. In general, the parameters χ_{ij} are different from each other, however in this study we consider two completely equivalent dots which have nearly the same Bohr radius of the excitons and transitional dipole moment. For the sake of simplicity, we assume the parameters $\chi_{ij} = \omega$ for $i = j$, which means the two quantum dots have the same transition frequency, and assume the positive real numbers $\chi_{ij} = g$ for $i \neq j$ which corresponds to the linear coupling constant of the two quantum dots. The parameters χ_{ijkl} of the non-linear terms are taken as positive constant, which means that no stable bi-exciton can be found in this system of the artificial molecule, and set to be $\chi_{ijkl} = \chi$. Under these assumptions, the Hamiltonian (1) can be simplified as following

$$H = \hbar\Omega\mathbf{N} + \hbar\chi\mathbf{N}^2 + \hbar\mathbf{G}(a_1^\dagger a_2 + a_2^\dagger a_1) + \hbar\chi(a_1^\dagger a_2 + a_2^\dagger a_1)^2, \qquad (2)$$

where $\Omega = \omega - 2\chi$, and $\mathbf{G} = g - 2\chi + 2\chi\mathbf{N}$. It is obvious that $\mathbf{N} = a_1^\dagger a_1 + a_2^\dagger a_2$ is a constant of motion, which means $[\mathbf{N}, H] = 0$ and the total exciton number of the coupled quantum dots is conservation quantity. In such a situation, we find that it is more convenient to give the solution of the Schrödinger equation governed by Hamiltonian (2) by virtue of Schwinger representation of the angular momentum [13]. That is, we can introduce the angular momentum operators

$$J_x = \frac{1}{2}(a_1^\dagger a_2 + a_2^\dagger a_1), \tag{3a}$$

$$J_y = \frac{1}{2i}(a_1^\dagger a_2 - a_2^\dagger a_1), \tag{3b}$$

$$J_z = \frac{1}{2}(a_1^\dagger a_1 - a_2^\dagger a_2) \tag{3c}$$

from the bosonic annihilation and creation operators of the two exciton modes. The operators of (3a-3c) satisfy the commutation relations for the Lie algebra of SU(2):

$$[J_i, J_j] = i\varepsilon_{ijk}J_k, \qquad i, j, k = x, y, z, \tag{4}$$

where the Levi-Cività tensor ε_{ijk} is equal to $+1$ and -1 for even and odd permutation of its indices, respectively, and zero otherwise. From (3a-3c), the total angular momentum operator can be expressed as followings

$$J^2 = \frac{\mathbf{N}}{2}(\frac{\mathbf{N}}{2} + 1). \tag{5}$$

In fact, $\mathbf{N}$ itself commutes with all the operators of Eqs. (3a-3c) and Eq.(5). For a fixed total excitonic number $\mathcal{N}$, The common eigenstates of J^2 and J_z are the two-mode Fock states

$$|jm\rangle = |m_1, m_2\rangle = \frac{(a_1^\dagger)^{j+m}(a_2^\dagger)^{j-m}}{\sqrt{(j+m)!(j-m)!}}|0\rangle \tag{6}$$

with eigenvalues $j = \mathcal{N}/2$ and $m = -\mathcal{N}/2, \cdots, \mathcal{N}/2$, where $|m_1, m_2\rangle$ is Fock state with $m_1 = j + m$ excitons and $m_2 = j - m$ excitons in dot A and dot B respectively. Although $j \pm m$ (m_1, m_2) must be integers, j and m can both be integers or both be half-odd integers. For consistency, all j are replaced by $\mathcal{N}/2$ in the following expressions

In terms of an $SO(3)$ rotation $e^{i\pi/2J_y}$ of $\hbar 2\mathbf{G}J_x + \hbar 4\chi J_x^2$, the Eq. (2) can be simplified into :

$$\begin{aligned} H &= \hbar\Omega\mathbf{N} + \hbar\chi\mathbf{N}^2 + \hbar 2\mathbf{G}J_x + \hbar 4\chi J_x^2 \\ &= \hbar\Omega\mathbf{N} + \hbar\chi\mathbf{N}^2 + \hbar 2\mathbf{G}e^{-i(\pi/2)J_y}J_z e^{i(\pi/2)J_y} \\ &+ \hbar 4\chi e^{-i(\pi/2)J_y}J_z^2 e^{i(\pi/2)J_y}. \end{aligned} \tag{7}$$

The eigenfunctions $\Psi_{\mathcal{N}/2,m}$ and the eigenvalues $E_{\mathcal{N}/2,m}$ of Hamiltonian (7) can be obtained very easily as

$$|\Psi_{\frac{\mathcal{N}}{2},m}\rangle = \sum_{m'=-\frac{\mathcal{N}}{2}}^{\frac{\mathcal{N}}{2}} \mathcal{D}^{\frac{\mathcal{N}}{2}}_{m'm}(\tfrac{\pi}{2})|\frac{\mathcal{N}}{2}, m'\rangle, \tag{8a}$$

$$E_{\mathcal{N}/2,m} = \hbar\Omega\mathcal{N} + \hbar\chi\mathcal{N}^2 + \hbar 2\mathcal{G}m + \hbar 4\chi m^2, \tag{8b}$$

where $\mathcal{N}$ denotes the total excitonic number of two dots and $\mathcal{G} = g - 2\chi + 2\chi\mathcal{N}$. We can obtain the Wigner's formula for $\mathcal{D}^{\frac{\mathcal{N}}{2}}_{m'm}(\frac{\pi}{2})$ as

$$\begin{aligned} \mathcal{D}^{\frac{\mathcal{N}}{2}}_{m'm}(\frac{\pi}{2}) &= \sum_k (-1)^{k-m-m'}(\tfrac{1}{2})^{\frac{\mathcal{N}}{2}} \\ &\times \frac{\sqrt{(\frac{\mathcal{N}}{2}+m)!(\frac{\mathcal{N}}{2}-m)!(\frac{\mathcal{N}}{2}+m')!(\frac{\mathcal{N}}{2}-m')!}}{(\frac{\mathcal{N}}{2}+m-k)!k!(\frac{\mathcal{N}}{2}-k-m')!(k-m+m')!}, \end{aligned} \tag{9}$$

where we take the sum over k such that none of the arguments of factorials in the denominator is negative. Then the evolution operators of the total system is written as

$$U(t) = e^{-itH/\hbar} = \sum_{\mathcal{N}=0}^{\infty}\sum_{m=-\frac{\mathcal{N}}{2}}^{\frac{\mathcal{N}}{2}} e^{-itE_{\frac{\mathcal{N}}{2},m}/\hbar}|\Psi_{\frac{\mathcal{N}}{2},m}\rangle\langle\Psi_{\frac{\mathcal{N}}{2},m}|. \tag{10}$$

The wave function of the system $|\Psi(t)\rangle$ associated with the initial condition $|\Psi(t=0)\rangle$ can be expressed as $|\Psi(t)\rangle = U(t)|\Psi(t=0)\rangle$, that is

$$\begin{aligned} |\Psi(t)\rangle = \sum_{\mathcal{N}=0}^{\infty}\sum_{m=-\mathcal{N}/2}^{m=\mathcal{N}/2} &\exp\left(\frac{E_{\frac{\mathcal{N}}{2},m}}{i\hbar}t\right) \\ &\times |\Psi_{\frac{\mathcal{N}}{2}m}\rangle\langle\Psi_{\frac{\mathcal{N}}{2}m}|\Psi(t=0)\rangle. \end{aligned} \tag{11}$$

These coefficients $\langle\Psi_{\frac{N}{2}m}|\Psi(t=0)\rangle$ are rotating matrix elements which can be determined by Wigner's formula. Eq. (11) is a basic equation which will be used in our further discussions. In the following sections, we will discuss the entanglement of two exciton modes.

III. ENTANGLEMENT OF THE EXCITONIC STATES

Quantum entanglement plays the key role in the quantum information and quantum computation. In general, for any pure composite state $|\psi(A,B)\rangle$ of a bipartite system whose state space is $H_A \otimes H_B$, the entanglement can be measured by von Neumann's entropy of any one reduced density operator $\rho_A = Tr_B(|\psi(A,B)\rangle\langle\psi(A,B)|)$ or $\rho_B = Tr_A(|\psi(A,B)\rangle\langle\psi(A,B)|)$, where the reduced density operator of system A is obtained by tracing out system B and that of system B by tracing out system A. The entropy of the entanglement $E(\rho)$ for the bipartite pure state $|\psi(A,B)\rangle$ is defined as [14]

$$\begin{aligned} E(\rho) &= -\mathrm{Tr}(\rho_A \ln \rho_A) = -\mathrm{Tr}(\rho_B \ln \rho_B) \\ &= -\sum_i (\lambda_i \ln \lambda_i). \end{aligned} \tag{12}$$

Where λ_i are the eigenvalues of either ρ_A or ρ_B, they form the (square of the) coefficients of the Schmidt decomposition of the bipartite pure state. That is the bipartite pure state $|\psi(A,B)\rangle$ can be expressed by a set of the bi-orthogonal vectors using the Schmidt decomposition as

$$|\psi(A,B)\rangle = \sum_i \sqrt{\lambda_i}|\alpha_i\rangle_A|\beta_i\rangle_B, \tag{13}$$

where we have chosen the phases of our basis states so that no phases appear in the coefficients λ_i in the sum of Eq. (13), and $\{|\alpha_i\rangle, i = 0\cdots\}$ and $\{|\beta_i\rangle, i = 0\cdots\}$ are orthonormal states of the two subsystems A and B respectively.

In our system, we assume, without loss of generality, that the total number of excitons in the whole system is fixed by the initially given condition, such as L. In such condition, the maximally entangled state of this artificial molecular system is

$$|M\rangle = \frac{1}{\sqrt{L+1}}\sum_{l=0}^{L}|L-l,l\rangle, \tag{14}$$

where $|L-l,l\rangle$ represents that there are $L-l$ excitons in the quantum dot A and l excitons in the quantum dot B. According to Eq. (12), the entangled entropy of the maximally entangled state (14) is $\ln(1+L)$.

In our paper we assume that quantum dot A is initially excited and there are L excitons in dot A, and no excitons in quantum dot B. So the modes of the two quantum dots are disentangled at initial time $t = 0$. That is, the state of the whole system is a tensor product of the states of two subsystems A and B, i.e. $|\Psi(t=0)\rangle = |L\rangle_A \otimes |0\rangle_B$ with the Schwinger realization of this initial state as $|\Psi(t=0)\rangle = |\frac{L}{2}, \frac{L}{2}\rangle$. The system and each mode of the artificial molecule are in pure states respectively and each entropy is zero.

It is well known that any pure state still keeps purity with the unitary time evolution, but it is not true for each subsystem. With the evolution of the time, the initial pure state of each subsystem can be transformed into mixed states respectively. The Von Neumann entropy $E(\rho)$ in the Eq. (12) is increased, and two subsystems entangle each other. The degree of entanglement between two subsystems in the artificial molecule at any time can be described using Eq. (12).

For the initial state $|\frac{L}{2}, \frac{L}{2}\rangle$ of the system, we can obtain the total wave function of the system from Eq. (11) as

$$|\Psi(t)\rangle = \sum_{l'=-L/2}^{l'=L/2} \exp\Big(\frac{E_{\frac{L}{2},l'}}{i\hbar}t\Big)|\Psi_{\frac{L}{2},l'}\rangle\langle\Psi_{\frac{L}{2},l'}|\frac{L}{2},\frac{L}{2}\rangle, \tag{15}$$

where the normalized coefficients $\langle\Psi_{\frac{L}{2},l'}|\frac{L}{2},\frac{L}{2}\rangle$ are determined by Eq. (9) and Eq. (11). We will use Eq.(15) to discuss the degree of the entanglement of the two subsystems with the following several concrete examples.

Firstly, we consider that there is initially one exciton in the quantum dot A i.e. $L = 1$. In this case we can obtain the wave function of the whole system from Eq. (15) with initial state $|\frac{1}{2}, \frac{1}{2}\rangle$ as

$$\begin{aligned}|\Psi(t)^{(1)}\rangle &= [\cos(gt)|\tfrac{1}{2},\tfrac{1}{2}\rangle - i\sin(gt)|\tfrac{1}{2},-\tfrac{1}{2}\rangle] \\ &= [\cos(gt)|1\rangle_A 0\rangle_B - i\sin(gt)|0\rangle_A 1\rangle_B],\end{aligned} \tag{16}$$

where we have omitted the global phase factor of time dependence $e^{-i(\Omega+2\chi)t}$. The entropy of entanglement can be calculated as

$$E^{(1)}(t) = -\cos^2(gt)\ln[\cos^2(gt)] - \sin^2(gt)\ln[\sin^2(gt)]. \tag{17}$$

We find in this case the entropy of the entanglement periodically evolves with zero values at times $gt = k\frac{\pi}{2}$ where k is an integer. The entropy of the entanglement reaches its maximum $\ln 2$ at times $gt = (2k+1)\frac{\pi}{4}$ with integer k, and the maximally entangled states of the artificial molecule system is

$$\begin{aligned}|\Psi(t)^{(1)}\rangle &= \frac{|\frac{1}{2},\frac{1}{2}\rangle + e^{-i2gt}|\frac{1}{2},-\frac{1}{2}\rangle}{\sqrt{2}} \\ &= \frac{|1\rangle_A|0\rangle_B + e^{-i2gt}|0\rangle_A|1\rangle_B}{\sqrt{2}}.\end{aligned} \tag{18}$$

If there are initially two excitons in quantum dot A, then the wave function of the whole system with initial condition $|\Psi(t=0)\rangle = |1,1\rangle$ becomes

$$\begin{aligned}|\Psi(t)^{(2)}\rangle &= \alpha_1|1,-1\rangle + \alpha_2|1,0\rangle + \alpha_3|1,1\rangle \\ &= \alpha_1|0\rangle_A|2\rangle_B + \alpha_2|1\rangle_A|1\rangle_B + \alpha_3|2\rangle_A|0\rangle_B\end{aligned} \tag{19a}$$

and

$$\alpha_1 = \frac{1}{4}[e^{i2gt} + e^{-i2(g+4\chi)t} - 2], \tag{19b}$$

$$\alpha_2 = \frac{\sqrt{2}}{4}[e^{-i2(g+4\chi)t} - e^{i2gt}], \tag{19c}$$

$$\alpha_3 = \frac{1}{4}[e^{i2gt} + e^{-i2(g+4\chi)t} + 2], \tag{19d}$$

where global phase factor of time dependence $e^{-i2(\Omega+2\chi)t}$ also has been neglected. We can obtain the entropy of the entanglement as

$$E^{(2)}(t) = -\sum_{i=1}^{3}|\alpha_i|^2\ln|\alpha_i|^2, \tag{20}$$

where $|\alpha_i\rangle$ determined by Eqs. (19b-19d). The time instants t when the system becomes maximally entangled can be obtained by setting $E^{(2)}(t) = \ln 3$. Fig. 1 plots the entropy of the entanglement as the function of gt with two sets different parameters ratio in which we can find the maximally entangled state between two dots with the

time evolution, the maximally entangled state of the system presents a periodical behavior. But when the second order coupling constant between excitons is weak, the period becomes longer. It is worth noting that when the second order coupling constant equal to zero, our model becomes similar to that of a beam splitter model. In such a case, we cannot obtain maximally entangled states. The above discussion clearly shows that the entanglement between two dots depends on both the time evolution and the parameters χ/g.

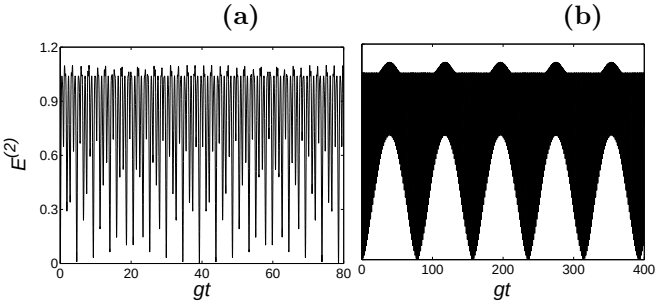

FIG. 1. $E^{(2)}$ is plotted as a function of gt for (a) $\chi/g = 0.34$ (b), $\chi/g = 0.01$.

For an arbitrary number of excitons L, the wave function of the whole system is described by Eq. (15). The entropy of the entanglement can be calculated as

$$E^{(L)}(t) = -\sum_{m'=-\frac{L}{2}}^{\frac{L}{2}} |\beta_{m'}|^2 \ln |\beta_{m'}|^2 \tag{21a}$$

with

$$\beta_{m'} = \sum_{m=\frac{L}{2}}^{\frac{L}{2}} \mathcal{D}^{\frac{L}{2}}_{\frac{L}{2},m}(\frac{\pi}{2}) e^{-\frac{i}{\hbar}E_{jm}t} \mathcal{D}^{\frac{L}{2}}_{m',m}(\frac{\pi}{2}), \tag{21b}$$

where $\mathcal{D}^{j}_{m',m}(\frac{\pi}{2})$ is determined by Eq. (9).

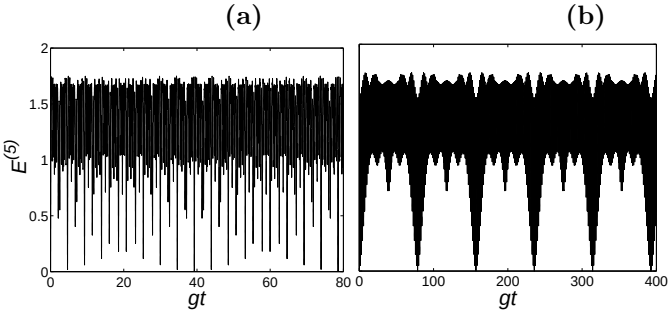

FIG. 2. $E^{(5)}$ is plotted as a function of gt for (a) $\chi/g = 0.34$ (b), $\chi/g = 0.01$.

In Fig. 2, the entropy of the entanglement is depicted as a function of gt when there are five excitons in the system of the coupled quantum dots. Numerical results show that when $L = 5$, we cannot find any time t that will generate maximally entangled states for any χ/g.

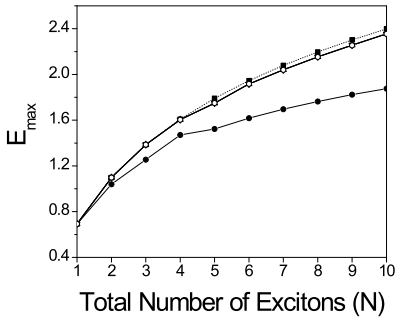

FIG. 3. Up to exciton number $N \leq 10$, the maximal values of the von Neumann entropy E_{max} are plotted as a function of exciton number N for $\chi/g = 0$(dots in the solid curve), $\chi/g = 0.01$, $\chi/g = 0.34$, $\chi/0.8$ (circles in solid curve). The values of the von Neumann entropy of the maximally entangled states are marked by squares in the dashing curve for different exciton number N.

We can use Eq. (21a-21b) to discuss the entanglement of the excitons between two subsystems A and B for any initially given exciton number in this artificial molecule. Here the entropy of the entanglement of two subsystem is discussed up to ten excitons. The numerical results show that we cannot obtain maximally entangled states when the total exciton number of the system is more than three for any parameters ratio χ/g and any time gt. But we can calculate maximal values E_{max} of the entropy of the entanglement for the two subsystems using eq.(21a-21b). Fig. 3 shows a comparison of the maximal values E_{max} of the entropy of the entanglement with the values of the entropy of the entanglement for the maximally entangled states up to the number of ten excitons for some parameter ratios $\chi/g = 0,\ 0.01,\ 0.34,\ 0.08$. From Fig.3, we know that the entropies of the entanglement for parameter ratio $\chi/g = 0$ for all exciton number is smaller than those for parameter ratios $\chi/g = 0.01,\ 0.34,\ 0.08$, which means our system can reach a larger entanglement than that of beam splitting model. At this point, the artificial molecule can be considered as a good entangled source with fixed exciton number than that of beam splitting model with the same photon number. For fixed exciton number, we find that the difference of the maximal values for the entropies of the entanglement between any two different parameter ratios among $\chi/g = 0.01,\ 0.34,\ 0.08$ is very small. Every circle in solid curve of Fig.3 actually denotes three maximal values of the entropies of the entanglement which cannot be resolved in that resolution of the figure.

IV. CONCLUSIONS

We discuss the entanglement of the excitonic states in the system of the coupled quantum dots (or artificial

molecule) with fixed total exciton number by the entropy of the entanglement. We find that when the total number of excitons for artificial molecule is more than three, we can not find the maximally entangled states, but numerical results show that the maximal values of the entropy of the entanglement for some parameter ratios is bigger than those of the beam splitting model from four to ten excitons. So this artificial molecule can be considered as a good entangled source.

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