Contents

1 Photon bunching and antibunching effects in non-stationary fields†

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1.1 INTRODUCTION

In the 1950s, Hanbury-Brown and Twiss [1] carried out the fundamental measurements of photon-number correlations demonstrating that photons of classical light exhibit a tendency to distribute themselves preferentially in bunches rather than at random. This effect was coined the *photon bunching* (PB). The Hanbury-Brown and Twiss experiments have triggered theoretical and experimental search for light exhibiting effect opposite to PB, i.e., the so-called *photon antibunching* (PAB). It was first observed in the process of resonance fluorescence from an atom by Kimble et al. [2] only twenty years after the first demonstration of PB [1]. Subsequent generations of PAB in resonance fluorescence were reported in Refs. [3]. Antibunched light was also generated in other processes, including parametric down-conversion [4], degenerate [5] and nondegenerate [6] parametric amplification, or destructive two-photon interference [7]. Analysis of PB and PAB effects in nonlinear optical systems has been one of the hot topics of quantum optics for several decades [8]–[11].

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It is a well known fact, that the PAB of stationary fields is one of the manifestations of nonclassical properties of light. PAB cannot be understood within the classical field theory describing light as a wave. But, on the other hand, it has a simple interpretation in particle (photon) models by the rise of the joint probability of two detected particles upon the increase of their time separation τ close to zero.

In this paper we would like to address the following question: How to describe PB and PAB in non-stationary fields in a way closest to the original photodetection interpretation of the effect in stationary-field regime, having a guarantee that the PAB cannot occur for classical light. PAB of non-stationary fields has already been analyzed theoretically in various nonlinear optical models (see, e.g., Refs. [12]–[17]). Here, we show that the approaches developed for stationary fields, when applied directly to analyze the non-stationary fields, are by no means unique and might lead to self-contradictory predictions [18]. And what is more counterintuitive, we will show that, in some cases, the standard definitions do not exclude the possibility of observation of the PAB artifacts in classical non-stationary fields [19].

The *classical light* is usually defined (see, e.g., Refs. [8, 9]) to be one for which the Glauber-Sudarshan P-function, i.e., the weight factor in the coherent-state representation of the density matrix

$$\widehat{\rho} = \int d^2\{\alpha_j\} P(\{\alpha_j\}) |\{\alpha_j\}\rangle\langle\{\alpha_j\}|, \tag{1.1}$$

is a probability distribution, i.e., is nonnegative and cannot be more singular than the Dirac δ-function. Otherwise, the state is nonclassical. In Eq. (1.1), the compact notation for the multimode field is used, where the argument $\{\alpha_j\}$ stands for $(\alpha_1, \alpha_2, ...)$.

To test various definitions of photon antibunching, we analyze a parametric frequency conversion – a process of exchanging photons between signal and idler optical modes of different frequencies. It is one of the most fundamental models in quantum optics both from theoretical and experimental viewpoints. The model has been successfully applied to describe various optical phenomena. In particular, wave mixing and beam splitting (see, e.g., Refs. [20, 9]), Raman scattering [9, 21], a two-level atom driven by a single mode electromagnetic field (e.g., Ref. [22]) or, by straightforward generalization, coherent or incoherent spontaneous emission from a system of N two-level atoms [23]. There have been great advances in the construction of frequency converters for over 40 years. The frequency conversion devices are based on the coupling of light waves in, e.g., nonlinear dielectric crystals such as KDP, $LiNbO_3$ or $LiIO_3$[24]. A simple quantum description of the parametric frequency conversion was given by Louisell [25]. The remarkable property of the Louisell model is the classical-like evolution or, explicitly, the conservation of quasidistributions along classical trajectories as was predicted by Glauber [26] and experimentally observed by Huang and Kumar [27] for initial quantum states. Our interest in the frequency conversion comes from this conservation of the initial (classical or quantum) character of the fields during the process.

This paper is organized as follows. In Sect. 1.2, we give a short account of the most popular definitions of PAB and we propose a generalized definition. In Sect. 1.3, we briefly review the parametric frequency converter model and Glauber's theorem useful for our analysis of PAB. In Sect. 1.4, we show discrepancies between three definitions of PAB for quantum non-stationary fields. In Sect. 1.5, we show that there are classical non-stationary fields exhibiting apparently PAB according to the standard definitions.

1.2 CRITERIA FOR PHOTON ANTIBUNCHING

The central role in definitions of PAB in a single-mode radiation field plays the intensity correlation function [28]

$$G^{(2)}(t,t+\tau) \;=\; \langle \mathcal{T} : \widehat{n}(t)\widehat{n}(t+\tau) : \rangle = (\alpha c S)^{-2} P_2(t,t+\tau), \qquad (1.2)$$

where $\widehat{n}(t)$ denotes the photon-number density operator, and products of the operators are written in normal order (: :) and in time order ($\mathcal{T}$). As was proved by Glauber [28], $G^{(2)}(t,t+\tau)$ is directly related to the joint detection probability, $P_2(t,t+\tau)$, of detecting two photons, one at time t and another at time $(t+\tau)$, by photodetector of quantum efficiency α with photocathode of area S. In Eq. (1.2), c denotes the light velocity; the space coordinates are suppressed and only one photodetector is assumed.

Different normalizations of $G^{(2)}(t,t+\tau)$ can be applied in the analysis of photon-number correlations. Here, we analyze the normalized two-time second-order intensity correlation functions defined as

$$\begin{aligned} g^{(2)}_{\mathrm{I}}(t,t+\tau) &= \frac{G^{(2)}(t,t+\tau)}{\left[G^{(1)}(t)\right]^2} \\ g^{(2)}_{\mathrm{II}}(t,t+\tau) &= \frac{G^{(2)}(t,t+\tau)}{G^{(1)}(t)G^{(1)}(t+\tau)} \\ g^{(2)}_{\mathrm{III}}(t,t+\tau) &= \frac{G^{(2)}(t,t+\tau)}{\sqrt{G^{(2)}(t,t)G^{(2)}(t+\tau,t+\tau)}}\,, \end{aligned} \qquad (1.3)$$

where $G^{(1)}(t) = \langle n(t)\rangle = \langle \widehat{a}^{\dagger}(t)\widehat{a}(t)\rangle$ is the light intensity. The *photon antibunching* according to the jth (j=I, II, III) definition occurs if the normalized intensity correlation function $g^{(2)}_j(t,t+\tau)$ increases from its initial value at $\tau=0$, i.e.,

$$\Delta g_j(t,t+\tau) \equiv g^{(2)}_j(t,t+\tau) - g^{(2)}_j(t,t) > 0. \qquad (1.4)$$

The *photon bunching* occurs for decreasing correlation function $g^{(2)}_j(t,t+\tau)$, whereas *photon unbunching* takes place if $g^{(2)}_j(t,t+\tau)$ is locally constant. Alternatively, on assumption that $g_j(t,t+\tau)$ is a well-behaved function of τ, the PAB according to the jth definition occurs if the lowest-order (say n_0) non-vanishing derivative of $g_j^{(2)}(t,t+\tau)$ [or $\Delta g_j(t,t+\tau)$] is positive at $\tau=0$, i.e., exists such $n_0 \geq 1$ that

$$\gamma_j(t) \equiv \gamma_j^{(n_0)}(t) = \left. \frac{\partial^{n_0}}{\partial \tau^{n_0}} g_j^{(2)}(t,t+\tau) \right|_{\tau=0} > 0 \tag{1.5}$$

if the derivatives $(\partial/\partial\tau)^n g_j^{(2)}(t,t+\tau)$ vanish at $\tau=0$ for $n=1,...,n_0-1$. The field exhibits PB if the lowest-order non-vanishing derivative, $\gamma_j(t)$, is negative. If the derivatives of all orders vanish, $\gamma_j(t)=0$, the field is said to be unbunched. In the Sects. 1.4 and 1.5, we will use both parameters $\gamma_j(t)$ and correlation functions $\Delta g_j(t,t+\tau)$ to analyze PB and PAB effects in a model of frequency conversion.

Both Def. I (see [9] and references therein) and Def. II (see, e.g., [10]) have been applied to analyze PAB of non-stationary light generated in various non-linear optical processes. In particular, analysis of PAB in *non-stationary* light has been studied by, e.g., Singh [16] and Feng et al. [17] with the help of Def. I, and by, e.g., Kryszewski and Chrostowski [12], Srinivasan and Udayabaskaran [13], Dung et al. [14], and Aliskenderov et al. [15] by applying Def. II.

For stationary fields, i.e., fields satisfying the property $G^{(2)}(t,t+\tau)=G^{(2)}(\tau)$, Defs. I–III are equivalent up to a τ-independent factor. In Sect. 1.4, we will show that the predictions of PAB according to Defs. I–III can be essentially different for non-stationary fields, even though they coincide in stationary fields. Differences between various approaches to PAB result from the normalization functions of $G^{(2)}(t,t+\tau)$, which for Def. I is independent of τ, whereas in cases of Defs. II and III, the normalizations are τ-dependent but in two not equivalent ways for non-stationary fields.

The Cauchy-Schwarz inequality,

$$\left[G^{(2)}(t,t+\tau)\right]^2 \leq G^{(2)}(t,t)G^{(2)}(t+\tau,t+\tau), \tag{1.6}$$

must be fulfilled for any classical field. Thus the violation of inequality (1.6) can reflect the corpuscular nature of light and can serve as a criterion of PAB. All definitions of the PAB effect for stationary fields are based on the Cauchy-Schwarz inequality. However, for non-stationary fields, PAB according to Defs. I and II does not imply violation of the Cauchy-Schwarz inequality (1.6). In Sect. 1.5, we will give examples of classical non-stationary fields exhibiting apparently PAB according to Defs. I and II. By contrast, PAB according to Def. III occurs for nonclassical fields only, independent of the stationary-field condition. This conclusion is readily obtained by comparing the form of the correlation functions $g_{\rm III}^{(2)}(t,t+\tau)$ with the Cauchy-Schwarz inequality, given by (1.6).

1.3 MODEL FOR TESTING PHOTON ANTIBUNCHING

We will test different approaches to PB and PAB in a process of parametric frequency conversion. The model can be described by the interaction Hamiltonian [25]:

$$\widehat{H}_{\rm int} = \hbar\kappa \widehat{a}_{\rm a}\widehat{a}_{\rm b}^{\dagger} \exp({\rm i}\Delta\omega t) + {\rm h.c.}, \tag{1.7}$$

where $\Delta\omega = \omega_{\rm L} + \omega_{\rm b} - \omega_{\rm a}$, and $\widehat{a}_{\rm a,b}$ are the annihilation operators for the signal (with subscript a) and idler (subscript b) modes; κ is the real coupling constant. For simplicity, we analyze only the resonance case for $\Delta\omega = 0$. One can interpret the process described by Eq. (1.7) as the conversion of frequency $\omega_{\rm a}$ to $\omega_{\rm b}$ assisted by intensive classical light of frequency $\omega_{\rm L}$. Thus, instead of exact quantum Hamiltonian describing the three-photon interaction, $\widehat{H}_{\rm int} = \hbar\kappa' \widehat{a}_{\rm L}\widehat{a}_{\rm a}\widehat{a}_{\rm b}^{\dagger} + {\rm h.c.}$, we use its approximation given by (1.7), where the amplitude of classical light is included in the coupling constant $\kappa = \kappa' \langle \widehat{a}_{\rm L} \rangle = \kappa' \alpha_{\rm L}$. Model given by Eq. (1.7) can be realized by a beam splitter.

The solutions of the Heisenberg equation of motion for the signal and idler modes are [25]

$$\widehat{a}_j(t) \quad = \quad \cos(\kappa t)\, \widehat{a}_j - {\rm i} \sin(\kappa t)\, \widehat{a}_k \ , \tag{1.8}$$

where $\widehat{a}_j \equiv \widehat{a}_j(0)$, $j = a, b$ and $k = b, a$, respectively. According to Glauber's theorem [26], the two-mode Glauber-Sudarshan P-function can be given by

$$P(\alpha_{\rm a}, \alpha_{\rm b}, t) = P\left\{\alpha_{\rm a}(-t), \alpha_{\rm b}(-t), 0\right\}, \tag{1.9}$$

where

$$\alpha_j(t) = \cos(\kappa t)\alpha_j - {\rm i}\sin(\kappa t)\alpha_k \tag{1.10}$$

are the solutions of classical equations of motion for the frequency converter [25]. The two-mode P-function remains constant along classical trajectories $\alpha_j(t)$. In other words, if both the signal and idler modes are initially quantum (classical), then they will preserve their original quantum (classical) character for the whole evolution. This property was experimentally verified by Huang and Kumar [27].

In the next sections, we will analyze PAB in quantum and, then, classical non-stationary fields generated by this model.

1.4 PHOTON CORRELATIONS IN QUANTUM FIELDS

Let us analyze the parametric frequency conversion of the signal and idler modes initially in Fock states with photon numbers $N_{\rm a}$ and $N_{\rm b}$, respectively. By applying Eq.

(1.9), we readily find the evolution of the two-mode Glauber-Sudarshan P-function

$$P(\alpha_\mathrm{a},\alpha_\mathrm{b},t)=\frac{1}{\pi^2}\prod_{j=\mathrm{a,b}}\frac{N_j!}{(2N_j)!}\frac{\exp(r_j^2)}{r_j}\left(\frac{\partial}{\partial r_j}\right)^{2N_j}\delta(r_j)\Bigg|_{r_j=|\alpha_j(-t)|} \qquad (1.11)$$

via derivatives of the Dirac δ-function of the classical solutions, given by Eq. (1.10). It is seen that the P-function remains singular evolving along classical trajectories. Thus, the field is nonclassical for arbitrary evolution times.

Table 1.1 All possible predictions of photon bunching and antibunching of *quantum* **fields according to Defs. I, II and III. Signal and idler modes are initially in Fock states with: (i) $N_\mathrm{a}=2$, $N_\mathrm{b}=1$ (marked by prime) and (ii) $N_\mathrm{a}=3$, $N_\mathrm{b}=1$ (double prime). Here, $f\{x\}\equiv\frac{1}{2}\arccos(x)$.**

case	*Def. I*	*Def. II*	*Def. III*	*examples*
1.	bunching	bunching	bunching	$\kappa t\in\left(\pi-f\{\frac{3}{5}\},\pi\right)'$
2.	**anti**bunching	bunching	bunching	$\kappa t\in\left(\pi-f\{\frac{1}{3}\},\pi\right)''$
3.	bunching	**anti**bunching	bunching	$\kappa t\in\left(0,f\{\frac{1}{3}\}\right)''$
4.	**anti**bunching	**anti**bunching	bunching	$\kappa t\in\left(0,f\{\frac{3}{5}\}\right)'$
5.	bunching	bunching	**anti**bunching	$\kappa t\in\left(f\{-\frac{1}{5}\},\frac{\pi}{2}\right)'$
				$\kappa t\in\left(\pi-f\{\frac{1}{3}\},\pi-f\{\frac{3}{5}\}\right)'$
				$\kappa t\in\left(f\{-\frac{1}{3}\},\frac{\pi}{2}\right)''$
6.	**anti**bunching	bunching	**anti**bunching	$\kappa t\in\left(\pi-f\{-\frac{1}{5}\},\pi-f\{\frac{1}{3}\}\right)'$
				$\kappa t\in\left(\pi-f\{-\frac{1}{3}\},\pi-f\{\frac{1}{3}\}\right)''$
7.	bunching	**anti**bunching	**anti**bunching	$\kappa t\in\left(f\{\frac{1}{3}\},f\{-\frac{1}{5}\}\right)'$
				$\kappa t\in\left(f\{\frac{1}{3}\},f\{-\frac{1}{3}\}\right)''$
8.	**anti**bunching	**anti**bunching	**anti**bunching	$\kappa t\in\left(f\{\frac{3}{5}\},f\{\frac{1}{3}\}\right)'$
				$\kappa t\in\left(\frac{\pi}{2},\pi-f\{-\frac{1}{5}\}\right)'$
				$\kappa t\in\left(\frac{\pi}{2},\pi-f\{-\frac{1}{3}\}\right)''$

Here, we analyze all cases for which the three definitions of PAB might not be equivalent for some evolution times. These cases are listed in table 1.1 with examples of the nonclassical signal fields presented graphically in Fig. 1.1 for the parameters γ_j and in Fig. 1.2 for the correlations Δg_j. We refer to these ordinal numbers of the cases throughout the paper. In particular, they are given in the upper part of the figures. We present correlation functions for the signal mode only. Thus, we can consequently omit subscript a in correlation functions: $G^{(2)}(t_1,t_2)\equiv G^{(2)}_\mathrm{a}(t_1,t_2)$, $g_j^{(2)}\equiv g_{j,\mathrm{a}}^{(2)}$, and $\Delta g_j\equiv\Delta g_{j,\mathrm{a}}$ for j=I, II, III. Due to the symmetry of the solutions (1.8) for $j=a,b$, one can deduce the explicit expressions for the idler mode simply by interchanging the subscripts. Exact analytical solutions for the normalized correlation functions $g_j^{(2)}(t,t+\tau)$ (j=I, II, III) were obtained in Ref. [18] for arbitrary initial

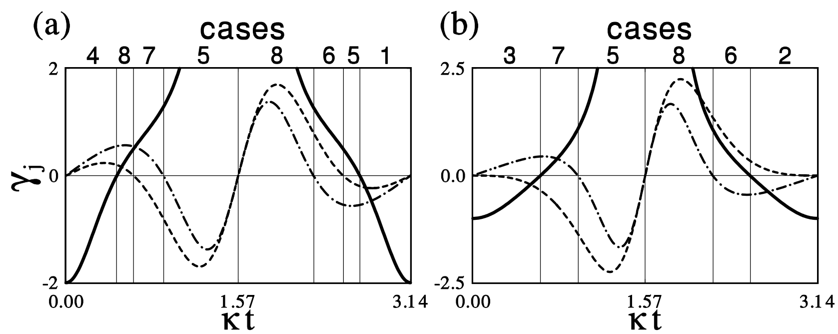

Fig. 1.1 Quantum-field evolution of the parameters $\gamma_{\rm I}(t)$ (dashed lines), $\gamma_{\rm II}(t)$ (dot-dash) and $\gamma_{\rm III}(t)$ (solid). Initially, both signal and idler modes are in Fock states with: **(a)** $N_{\rm a}=2$, $N_{\rm b}=1$ and **(b)** $N_{\rm a}=3$, $N_{\rm b}=1$.

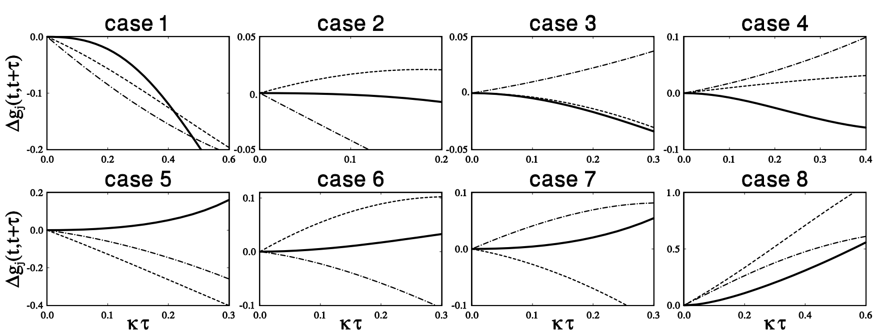

Fig. 1.2 Illustration of eight different predictions of photon antibunching of quantum fields, corresponding to the cases analyzed in Table 1.1 and Fig. 1.1. The two-time signal-mode correlation functions $\Delta g_{\rm I}(t,t{+}\tau)$ (dashed curves), $\Delta g_{\rm II}(t,t{+}\tau)$ (dot-dashed) and $\Delta g_{\rm III}(t,t{+}\tau)$ (solid) are plotted in their dependence on the re-scaled time separation $\kappa\tau$ for fixed values of the evolution time: (case 1) $\kappa t=2.8$, (2) $\kappa t=2.6$, (3) $\kappa t=0.1$, (4) $\kappa t=0.1$, (5) $\kappa t=1.0$, (6) $\kappa t=2.3$, (7) $\kappa t=0.7$, and (8) $\kappa t=1.8$. Signal and idler modes are initially in Fock states with $N_{\rm a}=3$ and $N_{\rm b}=1$ in cases 2 and 3, or with $N_{\rm a}=2$ and $N_{\rm b}=1$ in all other cases.

Fock states. However, for the purpose of our presentation, it is enough to analyze only two special cases.

If the initial signal mode is in Fock state with $N_{\rm a}=2$, and idler mode in Fock state with $N_{\rm b}=1$, the Taylor expansions of the correlation functions $\Delta g_j(t,t+\tau)$ are

$$\Delta g_{\rm I}(t,t+\tau) \quad = \quad \frac{-1+3\cos(2\kappa t)}{\langle n_{\rm a}(t)\rangle^2}\sin(2\kappa t)(\kappa\tau)+\mathcal{O}(\tau^2)$$

$$\begin{aligned}
\Delta g_{\mathbb{II}}(t,t+\tau) &= \frac{1+5\cos(2\kappa t)}{\langle n_{\rm a}(t)\rangle^3}\sin(2\kappa t)(\kappa\tau)+\mathcal{O}(\tau^2) \\
\Delta g_{\mathbb{III}}(t,t+\tau) &= 2\sec^2(\kappa t)\frac{3-5\cos(2\kappa t)}{[5-3\cos(2\kappa t)]^2}(\kappa\tau)^2+\mathcal{O}(\tau^3), \qquad (1.12)
\end{aligned}$$

where the mean photon number is $\langle n_{\rm a}(t)\rangle = \frac{1}{2}[3+\cos(2\kappa t)]$; and $\mathcal{O}(\tau^k) \equiv \mathcal{O}(\{\kappa\tau\}^k)$ denotes the order of magnitude. The discrepancies between Defs. I, II, and III are well pronounced both analytically and graphically in Fig. 1.1(a) with the help of the parameters γ_j and in Fig. 1.2 directly in terms of the correlation functions $\Delta g_j(t,t+\tau)$ (j=1, 2, 3). During the evolution of initial Fock states $|N_{\rm a},N_{\rm b}\rangle = |2,1\rangle$ almost all (except cases 2 and 3) are observed. The remaining two cases can be found, e.g., in the signal-field evolution of the initial Fock states $|N_{\rm a},N_{\rm b}\rangle = |3,1\rangle$. Here, we obtain

$$\begin{aligned}
\Delta g_{\rm I}(t,t+\tau) &= -\frac{6\sin^2(\kappa t)}{\langle n_{\rm a}(t)\rangle^2}\sin(2\kappa t)(\kappa\tau)+\mathcal{O}(\tau^2) \\
\Delta g_{\mathbb{II}}(t,t+\tau) &= 3\frac{1+3\cos(2\kappa t)}{\langle n_{\rm a}(t)\rangle^3}\sin(2\kappa t)(\kappa\tau)+\mathcal{O}(\tau^2) \\
\Delta g_{\mathbb{III}}(t,t+\tau) &= \frac{1-3\cos(2\kappa t)}{[3-\cos(2\kappa t)]^2}\sec^2(\kappa t)\,(\kappa\tau)^2+\mathcal{O}(\tau^3), \qquad (1.13)
\end{aligned}$$

where $\langle n_{\rm a}(t)\rangle = 2+\cos(2\kappa t)$. The evolution of the parameters γ_j, given by the expansion coefficients in (1.13) are presented in Fig. 1.1(b). We find six out of eight different predictions, including cases 2 and 3 not observed in the evolution of $|N_{\rm a},N_{\rm b}\rangle = |2,1\rangle$. The latter two cases are also presented in Fig. 1.2 in a standard way for the correlation functions evolving with the time separation τ for fixed values of time t. The values of evolution times t given in table 1.1 are calculated from (1.12)–(1.13).

In conclusion, during the evolution of the nonclassical signal field in the parametric frequency converter, one observes that both PB and PAB effects from Defs. I–III can be accompanied, for some evolution times, with the same or different correlations of photons derived from other two definitions. We have given examples of all these cases in Figs. 1.1 and 1.2, and table 1.1.

1.5 PHOTON CORRELATIONS IN CLASSICAL FIELDS

If the initial modes are in a superposition of coherent states (with amplitudes α_{j0}, where j =a,b) and chaotic fields (with intensities $\langle n_{{\rm ch},j}\rangle$), then the evolution of the frequency converter is described by the following Glauber-Sudarshan P-function

$$P(\alpha_{\rm a},\alpha_{\rm b},t) = \frac{1}{\pi^2}\prod_{j={\rm a,b}}\frac{1}{\langle n_{{\rm ch},j}\rangle}\exp\left(-\frac{|\alpha_j(-t)-\alpha_{j0}|^2}{\langle n_{{\rm ch},j}\rangle}\right) \qquad (1.14)$$

evolving along the classical solutions, given by Eq. (1.10). The P-function (1.14) is a product of regular and positive Gaussian functions, thus describing explicitly the classical behavior of the idler and signal fields during the whole process of frequency conversion. Here, we analyze two special cases of these classical fields.

Table 1.2 Same as Table 1.1, but examples are given for *classical* fields. Signal and idler modes are initially described by Eq. (1.16) (marked by prime) or Eq. (1.18) (double prime).

case	Δg_{I} (*Def. I*)	Δg_{II} (*Def. II*)	Δg_{III} (*Def. III*)	*examples*
1.	negative (bunching)	negative (bunching)	negative (bunching)	$\kappa t \in \left(\frac{\pi}{2}, \pi\right)''$
2.	positive	negative (bunching)	negative (bunching)	$\kappa t \in \left(\frac{\pi}{2}, \pi\right)'$
3.	negative (bunching)	positive	negative (bunching)	$\kappa t \in \left(0, \frac{\pi}{2}\right)'$
4.	positive	positive	negative (bunching)	$\kappa t \in \left(0, \frac{\pi}{2}\right)''$
5.	negative (bunching)	negative (bunching)	positive	forbidden
6.	positive	negative (bunching)	positive	forbidden
7.	negative (bunching)	positive	positive	forbidden
8.	positive	positive	positive	forbidden

First, for simplicity, we assume that the mean photon numbers of chaotic photons in both modes are the same $\langle n_{\mathrm{ch,a}}\rangle = \langle n_{\mathrm{ch,b}}\rangle \equiv \langle n_{\mathrm{ch}}\rangle$ and the initial coherent amplitudes α_{j0} are real. We find

$$\begin{aligned}
\Delta g_{\mathrm{I}}(t,t+\tau) &= -2\langle n_{\mathrm{a}}(t)\rangle^{-2}\big\{\langle n_{\mathrm{ch}}\rangle + \langle n_a(t)\rangle\big\}N_-\sin(2\kappa t)\,(\kappa\tau) + \mathcal{O}(\tau^2) \\
\Delta g_{\mathrm{II}}(t,t+\tau) &= 2\langle n_{\mathrm{a}}(t)\rangle^{-3}\langle n_{\mathrm{ch}}\rangle\langle n_{\mathrm{coh,a}}(t)\rangle N_-\sin(2\kappa t)(\kappa\tau) + \mathcal{O}(\tau^2) \\
\Delta g_{\mathrm{III}}(t,t+\tau) &= -\Big\{G^{(2)}(t,t)(\langle n_{\mathrm{ch}}\rangle + 2N_+) - 4N_-^2\langle n_{\mathrm{ch}}\rangle\sin^2(2\kappa t)\Big\} \\
&\qquad \times\langle n_{\mathrm{ch}}\rangle[G^{(2)}(t,t)]^{-2}(\kappa\tau)^2 + \mathcal{O}(\tau^3) \le 0, \qquad (1.15)
\end{aligned}$$

where $N_\pm = \frac{1}{2}(\alpha_{\mathrm{a0}}^2 \pm \alpha_{\mathrm{b0}}^2)$. The time-dependent mean intensity of the signal mode is given by $\langle n_{\mathrm{a}}(t)\rangle = \langle n_{\mathrm{ch}}\rangle + \langle n_{\mathrm{coh,a}}(t)\rangle$, where $\langle n_{\mathrm{coh,a}}(t)\rangle = \alpha_{\mathrm{a0}}^2\cos^2(\kappa t) + \alpha_{\mathrm{b0}}^2\sin^2(\kappa t)$ is the time-dependent intensity of input coherent fields, and $\langle n_{\mathrm{ch}}\rangle$ is the initial intensity of chaotic field. Moreover, $G^{(2)}(t,t) = \langle n_{\mathrm{coh,a}}(t)\rangle^2 + 4\langle n_{\mathrm{ch}}\rangle\langle n_{\mathrm{coh,a}}(t)\rangle + 2\langle n_{\mathrm{ch}}\rangle^2$ is the single-time correlation function. In expansion (1.15), similarly to Eq. (1.13), the first τ-derivative of $g_{\mathrm{III}}^{(2)}(t,t+\tau)$ vanishes at $\tau = 0$. Expansions (1.15) have simple interpretation. The correlation function $\Delta g_{\mathrm{III}}(t,t+\tau)$ is never positive, thus PAB according to Def. III cannot be observed. On the contrary, both $\Delta g_{\mathrm{I}}(t,t+\tau)$ and $\Delta g_{\mathrm{II}}(t,t+\tau)$ oscillate between negative and positive values, therefore PAB according to Defs. I and II is apparently not prohibited. Surprisingly, predictions of Defs. I and II are opposite. As comes from Eq. (1.15), $\Delta g_{\mathrm{I}}(t,t+\tau)$ and $\Delta g_{\mathrm{II}}(t,t+\tau)$ have opposite signs and the same time-dependent function. Our conclusion is supported by graphical representations of the parameters

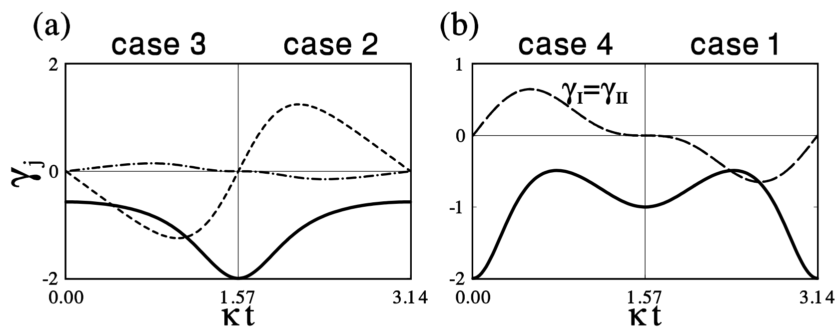

Fig. 1.3 Classical-field evolution of the parameters $\gamma_j(t)$ for initial superpositions of coherent state with thermal field: **(a)** $\rho_1(0)$ and **(b)** $\rho_2(0)$ given by Eqs. (1.16) and (1.18), respectively. Labels are the same as in Fig. 1.1.

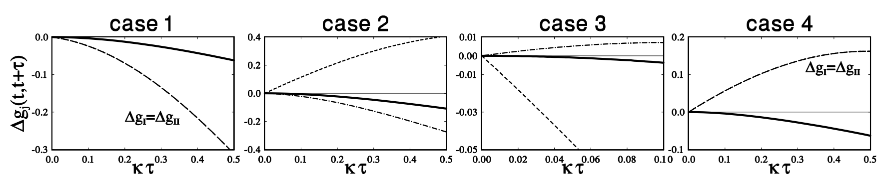

Fig. 1.4 Illustration of all four possible different predictions of PB and the PAB artifacts for classical fields as listed in Table 1.1. Initial conditions are given by Eqs. (1.16) and (1.18): (case 1) $\rho_2(0)$ at $\kappa t = 2$; (case 2) $\rho_1(0)$ at $\kappa t = 2$; (case 3) $\rho_1(0)$ at $\kappa t = 0.6$, and (case 4) $\rho_2(0)$ at $\kappa t = 0.4$. Evolution times κt are chosen with the help of Fig. 1.3. Labels and notation are the same as in Fig. 1.2.

γ_j in Fig. 1.3(a) and $\Delta g_j(t, t+\tau)$ in Fig. 1.4 (cases 2 and 3) for the initial condition

$$\rho_1(0) = \rho\big\{\alpha_{\rm a}^2 = \langle n_{\rm ch,a}\rangle = \langle n_{\rm ch,b}\rangle = 1, \alpha_{\rm b} = 0, t = 0\big\}. \tag{1.16}$$

Whenever PB is predicted according to one of Defs. I–II, it must be accompanied by the classical-field PAB artifact according to the other.

As the second example, we analyze another special case of the field (1.14), evolving in a way opposite the field evolution under initial condition (1.16). Let the signal mode is initially coherent (with real amplitude $\alpha_{\rm a0}$), whereas the idler mode is chaotic (with the mean photon number $\langle n_{\rm ch,b}\rangle$). We obtain the following power expansions of the normalized correlations $\Delta g_j(t, t+\tau)$ are

$$\begin{aligned}
\Delta g_{\rm I}(t,t+\tau) &= 2\big\{[2y\cot(\kappa t) - x\tan(\kappa t)]\langle n_{\rm a}(t)\rangle - xy\tan(\kappa t)\big\} \\
&\qquad \times \langle n_{\rm a}(t)\rangle^{-2}\,(\kappa\tau) + \mathcal{O}(\tau^2) \\
\Delta g_{\rm II}(t,t+\tau) &= 4x^2 y\csc(2\kappa t)\langle n_{\rm a}(t)\rangle^{-3}\,(\kappa\tau) + \mathcal{O}(\tau^2) \\
\Delta g_{\rm III}(t,t+\tau) &= -\frac{2\alpha_{\rm a0}^2\langle n_{\rm ch,b}\rangle(x^2+2y^2)}{(x^2+4xy+2y^2)^2}\,(\kappa\tau)^2 + \mathcal{O}(\tau^3) \le 0
\end{aligned} \tag{1.17}$$

in terms of the mean signal-mode intensity, $\langle n_{\rm a}(t)\rangle = x+y$, where $x=\alpha_{\rm a0}^2\cos^2(\kappa t)$ and $y=\langle n_{\rm a}(t)\rangle - x$. The short-time solution (1.17) reveals non-positive character of $\Delta g_{\rm III}(t,t+\tau)$, thus excluding the possibility of PAB according to Def. III. By contrast, both $\Delta g_{\rm I}(t,t+\tau)$ and $\Delta g_{\rm II}(t,t+\tau)$ change their signs during evolution. On further assumption of equal initial intensities of the signal and idler modes, namely

$$\rho_2(0)=\rho\big\{\alpha_{\rm a}^2=\langle n_{\rm ch,b}\rangle>0,\alpha_{\rm b}=\langle n_{\rm ch,a}\rangle=0,t=0\big\}, \tag{1.18}$$

we find that the normalized correlation functions $g_{\rm I}^{(2)}(t_1,t_2) = g_{\rm II}^{(2)}(t_1,t_2)$, and $g_{\rm III}^{(2)}(t_1,t_2)$ are independent of the initial intensities. Eqs. (1.17) reduce to

$$\Delta g_{\rm I}(t,t+\tau)=\Delta g_{\rm II}(t,t+\tau)=\cos^2(\kappa t)\sin(2\kappa t)\,(\kappa\tau)+\mathcal{O}(\tau^2) \tag{1.19}$$

$$\Delta g_{\rm III}(t,t+\tau)=-\frac{1+4\sin^2(\kappa t)+3\cos^2(2\kappa t)}{2[2-\cos^4(\kappa t)]^2}(\kappa\tau)^2+\mathcal{O}(\tau^3)\leq 0. \tag{1.20}$$

Evidently, the solution (1.19) takes positive values at some evolution times. We conclude that the classical-field PAB artifact according to Def. I occurs whenever it exists according to Def. II for the signal under the initial condition (1.18). These results are graphically represented in Fig. 1.3(b) and Fig. 1.4 (cases 1 and 4). It is worth comparing solution (1.19) with Eqs. (1.15) for $\Delta g_{\rm I}(t,t+\tau)$ and $\Delta g_{\rm II}(t,t+\tau)$, describing their opposite (out-of-phase) behavior (see Fig. 1.3(a)).

Table 1.2 summarizes our investigations of PB effects in classical fields. By virtue of the Cauchy-Schwarz inequality, PAB according to Def. III cannot occur for classical fields, thus the cases 5–7 in table 1.2 are excluded. However, the remaining cases 1–4 are observed in the evolution of classical fields as presented in Figs. 1.3 and 1.4. The classical PAB apparently exists according to both Defs. I and II.

PAB of classical fields can only be an artifact. So, it seems necessary to modify the conventional definitions in non-stationary regime. For instance, one can add an extra condition, which guarantees the nonclassical character of the field but keeping the original inequalities unchanged. Nevertheless, the problem of the unique description of PAB in non-stationary case would remain in the conventional definitions. On the contrary, these problems do not arise in the generalized approach to PAB (Def. III), where the Cauchy-Schwarz inequality is applied directly without any further assumptions.

1.6 CONCLUSION

We have generalized the concept of PB and PAB to describe non-stationary fields. The definition is based on the two-time second-order intensity correlation function $G^{(2)}(t_1,t_2)$ normalized by the square root of single-time second-order intensity correlations at two moments t_1 and t_2. This is contrary to the standard approaches to PAB, where the two-time correlation $G^{(2)}(t_1,t_2)$ is normalized either (i) by the single-time first-order intensity correlations at two moments, or (ii) by functions independent of the time separation $\tau = t_2 - t_1$. In a special case, when a field is stationary the generalized definition is equivalent to the standard definitions. However, as we have shown, the predictions of PAB according to these three approaches might be different for non-stationary fields. As an example, we have analyzed evolution of the signal mode during the parametric frequency conversion of initial Fock states and have found all (i.e., eight) possible different cases, when PAB (and also PB) effect according to one definition can be accompanied by arbitrary photon-correlation effects according to other two definitions. One may conclude, that the three definitions describe distinct quantum non-stationary phenomena.

The generalized definition of PAB was proposed on the basis of the Cauchy-Schwarz inequality without any assumptions concerning properties of the fields. Whereas the standard definitions come from the Cauchy-Schwarz inequality under stationary-field condition. Thus, PAB according to the generalized definition cannot occur for classical fields. However, as we have shown in the parametric frequency converter with classical initial conditions, the classical non-stationary fields possibly exhibit PAB artifacts according to the standard definitions without violating any classical inequalities.

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