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Realization of symmetric sharing of entanglement in semiconductor microcrystallites coupled by a cavity field

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The entanglement of excitonic states in a system of N spatially separated semiconductor microcrystallites is investigated. The interaction among the different microcrystallites is mediated by a single-mode cavity field. It is found that the symmetric sharing of the entanglement (measured by the concurrence) between any pair of the excitonic state with N qubits defined by the number states (vacuum and a single-exciton states) or the coherent states (odd and even coherent states) can be prepared by the cavity field for this system.

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I. INTRODUCTION

Quantum entanglement plays an important role in the quantum communication and quantum-information processing. One can implement quantum teleportation of a given state from one place to another by virtue of the entangled state [1,2]. Entanglement among many particles is essential for most quantum communication schemes. The simplest generalization of the entangled states for more than two particles is the so-called Greenberger-Horne-Zeilinger (GHZ) state [3]. The entanglement of the nearest-neighbor pairs for an infinite collection of qubits arranged in a line was studied by Wootters [4]. For a finite system, Koashi *et al.* [5] investigated the completely symmetric sharing of entanglement for an arbitrary pair of N qubits. Dür studied not only the symmetric sharing of the pairwise entanglement but also the nonsymmetric sharing in a system of N qubits [6]. The nearest-neighbor entanglement of N qubits in a ring configuration was also studied and further a concrete physical system of N spin-$\frac{1}{2}$ particles interacting via the Heisenberg antiferromagnetic Hamiltonian was given [7]. The question arises whether it is possible or not to prepare states with the symmetric sharing of entanglement in some real systems within the present-day technology. And, if yes, how to achieve such entangled states?

Any many-body system with defined qubits, if set in a properly chosen state, will evolve through states containing entangled qubits. Similarly, most of the ground states of real systems contain entangled states. However, for the purposes of quantum computation and quantum information, the most important aspects of quantum entanglement are especially (i) deterministic control over the quantum coherence of states and (ii) time evolution and occurrence of maximally entangled states. In this study, we focus on the latter topic, specifically, on the generation of the maximal pairwise entanglement. As was shown by Koashi *et al.* [5] that entanglement cannot be unlimitedly shared among an arbitrary number of qubits, and the degree of bipartite entanglement decreases with the increasing number of entangled pairs in an N-qubit system in which any pair of particles is entangled. The maximum degree of bipartite entanglement, measured in the concurrence, between any pair of qubits is bound by $2/N$. Here, we will investigate a physical realization of this maximally possible bipartite entanglement.

Within the past few years, advances in microfabrication technology have allowed researchers to create unique quantum confinement, and thereby have opened up a new realm of fundamental physics. As low-dimensional semiconductor structures, quantum dots attract a considerable interest because of their atomlike properties. They can lead to novel optoelectronic devices that can be applied to the emerging fields of quantum computing [8,9] and quantum-information processing [10,11]. It is well known that Coulomb-correlated electron-hole pairs called excitons can be optically generated and controlled in a single dot [12], and thus can be used to store the quantum information and realize quantum computing [13]. On the other hand, a significant fraction of quantum computing and information schemes relies on the strong-coupling regime of the cavity quantum electrodynamics (QED). The observed Rabi oscillations of excitons in a single quantum dot [14] suggest the possibility that the quantum dot cavity QED will be realized in the near future. However, an essential feature of a quantum dot is that the electronic energy levels are completely quantized, so the behavior of excitons deviates from the bosonic statistics. In the present paper, we will consider some slightly bigger semiconductor microstructures, such as the microcrystallites. In this case, the area of the microcrystallite is larger than that of the Bohr radius of the exciton, and the behavior of the excitons with low excitation are the same as that of the bosonic particles. Chuang *et al.* [15] showed that the quantum code of the bosonic mode enables a more efficient error correction. So, the mode of the excitons offers a possible physical implementation for such bosonic-mode coding. We propose a possible scheme to prepare the entangled excitonic states for the symmetric sharing in the system of N microcrystallites by virtue of the cavity QED. The cavity field mediates the interaction among semiconductor microcrystallites, and then the entangled excitonic states can be prepared by the cavity field.

We organize our paper as follows. In Sec. II, we will propose a scheme based on the present-day technology and model the Hamiltonian of the whole system. The solutions corresponding to this Hamiltonian are given for the general initial state. In Secs. III and IV, the bosonic exciton operator is used as an approach to deal with qubits uniformly. We will show how to prepare the entangled excitonic states with qubits defined by different excitonic states using various initial conditions of the cavity field. Any physical system cannot be isolated from its environment. The interaction between the system and the environment will result in their entanglement, then coherence of the qubit is destroyed with the time evolution. So, in Sec. V, we will demonstrate the environment effect on the entangled states. Finally, some comments and conclusions will be given.

II. MODEL AND ITS SOLUTION

We assume that there are N spatially separated semiconductor microcrystallites (also called large semiconductor quantum dots [16–18]) which are placed into an ideal semiconductor microcavity with a single-mode field, for example, the microcrystallites are embedded in a disk structure of the semiconductor, which is similar to the Imamoglu model for quantum dots [9]. And we assume that the radius R of each microcrystallite is much larger than the Bohr radius a_B of excitons, but smaller than the wavelength λ of the cavity field, that is, $a_B \ll R \leqslant \lambda$. Also the distance between each pair of microcrystallites is larger than the optical wavelength λ of the cavity field, and the microcrystallites indirectly interact by virtue of the cavity field. We also assume that there are few electrons excited from the valence band to the conduction band such that the exciton density for each microcrystallite is much smaller than the Mott density. So, all nonlinear terms included in the interaction of the exciton-exciton and exciton-photon can be neglected in our model, and the excitons are considered as ideal bosons. The cavity field is assumed to resonantly interact with the zero-momentum excitons in each microcrystallite, the thermalization of the excitons is neglected. Under the above conditions, we can use the effective Hamiltonian under the rotating wave approximation as follows [16,17]:

$$H=\hbar\omega a^\dagger a+\hbar\omega\sum_{j=1}^{N} b_j^\dagger b_j+\hbar\sum_{j=1}^{N} g_j(a^\dagger b_j+a b_j^\dagger), \quad (1)$$

where $a(a^\dagger)$ is the annihilation (creation) operator of the cavity field with frequency ω, and $b_j(b_j^\dagger)$ is the annihilation (creation) operator of the excitons in the jth microcrystallite with the same frequency ω as that of the cavity field. First, we assume that the coupling constants g_j with $j=1,2,\ldots,N$ between the cavity field and microcrystallites are different. We can give the Heisenberg equations of motion for the operators of the cavity field and the excitons as follows:

$$\frac{\partial A(t)}{\partial t}=-i\sum_j g_j B_j(t), \quad (2a)$$

$$\frac{\partial B_j(t)}{\partial t}=-ig_jA(t) \quad (j=1,2,\ldots,N), \quad (2b)$$

where the transformations $a(t)=A(t)e^{-i\omega t}$ and $b_j(t)=B_j(t)e^{-i\omega t}$ are applied. The solutions of Eqs. (2a)–(2b) can be obtained as

$$A(t)=a(0)\cos(G't)-i\sum_j f_j b_j(0), \quad (3a)$$

$$B_j(t)=\sum_m \left\{\delta_{jm}+\frac{g_j g_m[\cos(G't)-1]}{G'^2}\right\} b_m(0)-if_j a(0), \quad (3b)$$

where $G'\equiv\sqrt{\Sigma_{j=1}^N g_j^2}$; $f_j\equiv f_j(t)=g_j\sin(G't)/G'$, $a(0)$ and $b_j(0)$ $(j=1,\ldots,N)$ are the initial operators of the cavity field and excitons, respectively. We assume that the initial state of the whole system is $|\Psi(0)\rangle=|\psi(0)\rangle_C|0\rangle^{\otimes N}$, which means that the cavity field is initially in the state $|\psi(0)\rangle_C$, but there is no exciton in any microcrystallite. Then we can obtain the wave function as follows:

$$|\Psi(t)\rangle=U(t)|\psi(0)\rangle_C|0\rangle^{\otimes N}, \quad (4)$$

with the time-evolution operator $U(t)=e^{-iHt/\hbar}$.

III. PREPARATION OF THE ENTANGLED EXCITONIC STATE BY THE SINGLE-PHOTON STATE

It is well known that N qubits can be defined by the states of N spatially separated microcrystallites. The two most interesting states for both experimentalists and theoreticians are the no-exciton and one-exciton states denoted by $|0\rangle$ and $|1\rangle$, respectively. So, we choose the computational basis states of the qubit as $\{|0\rangle,|1\rangle\}$ for each microcrystallite. If the cavity field is initially in the single-photon state $|\psi(0)\rangle_C=a^\dagger|0\rangle_C$, which now can successfully be prepared by the experiment, and no exciton is initially in any microcrystallite, then $|\Psi(0)\rangle=a^\dagger|0\rangle_C|0\rangle^{\otimes N}$. Based on this initial condition, we interpolate the unit operator $U^\dagger(t)U(t)$ into Eq. (4) and consider the properties of the time-evolution operator $U^\dagger(t)OU(t)=O(t)$ and $U(t)|0\rangle=|0\rangle$, the wave function of the whole system can be obtained as follows:

$$\begin{aligned}|\Psi(t)\rangle&=U(t)a^\dagger|0\rangle_C|0\rangle^{\otimes N}\\&=a^\dagger(-t)|0\rangle_C|0\rangle^{\otimes N}\\&=\left[a^\dagger(0)\cos(G't)-i\sum_j f_j b_j^\dagger(0)\right]e^{-i\omega t}|0\rangle_C|0\rangle^{\otimes N}\\&=-ie^{-i\omega t}|0\rangle_C\sum_j f_j|1\rangle_j|0\rangle^{\otimes(N-1)}\\&\quad+e^{-i\omega t}\cos(G't)|1\rangle_C|0\rangle^{\otimes N},\end{aligned} \quad (5)$$

which has been returned into the original frame, and $|1\rangle_j|0\rangle^{\otimes(N-1)}$ means that $N-1$ microcrystallites have no excitons, and only one exciton is excited by the cavity field in the jth microcrystallite. We are interested in the entanglement between two subsystems of the excitons, such as, the nth and mth microcrystallites, then after tracing out the cavity field and the degrees of freedom of other $N-2$ microcrystallites, the reduced density operator for this pair of qubits can be obtained as

$$\rho(t)=f_m^2|10\rangle\langle 10|+f_m f_n|10\rangle\langle 01|+f_n^2|01\rangle\langle 01|+f_m f_n|01\rangle \times\langle 10|+\left\{\cos^2(G't)+\sum_{l\neq\{n,m\}} f_l^2\right\}|00\rangle\langle 00|. \tag{6}$$

The entanglement between two qubits can mathematically be described by using the concurrence [19]. We assume a pair of qubits whose density matrix is ρ_{12}. Then the concurrence of the density matrix ρ_{12} is defined as

$$C=\max\{\lambda_1-\lambda_2-\lambda_3-\lambda_4,0\}, \tag{7}$$

where λ_1, λ_2, λ_3, and λ_4, given in decreasing order, are the square roots of eigenvalues for the matrix

$$M_{12}=\rho_{12}(\sigma_{1y}\otimes\sigma_{2y})\rho_{12}^*(\sigma_{1y}\otimes\sigma_{2y}), \tag{8}$$

with the Pauli matrix

$$\sigma_{1y}=\sigma_{2y}=\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix},$$

where the asterisk denotes complex conjugation in the standard basis $\{|00\rangle,|01\rangle,|10\rangle,|11\rangle\}$, and σ_{1y} and σ_{2y} are expressed in the same basis. The entanglement of formation is a monotonically increasing function of C; and $C=0$ ($C=1$) corresponds to an unentangled state (maximally entangled state). The concurrence for the reduced density operator (6) can be obtained using Eqs. (7) and (8) as follows

$$C(t)=2f_m f_n=2g_m g_n\frac{\sin^2(G't)}{G'^2}. \tag{9}$$

It is found that the concurrence C periodically reaches its maximum value, but the values of the concurrences are different for different pairs, which means that the entanglements between different pairs are different. The coupling constants between the cavity field and microcrystallites determine the entanglement of each pair. So, we can realize symmetric sharing of entanglement of excitonic states in semiconductor microcrystallites only when all microcrystallites have the same interaction with the cavity field, e.g., $g_1=g_2=\cdots=g_N=g$, which may be obtained with the development of the microfabrication technology in the near future. Under this condition we can obtain the concurrence as

$$C(t)=\frac{2}{N}(1-\langle a^\dagger a\rangle)=\frac{2}{N}\sin^2(Gt), \tag{10}$$

with $G=g\sqrt{N}$, and the concurrence C periodically reaches its maximum value of $2/N$. Comparing the time evolution of the concurrence and the average photon number $\langle a^\dagger a\rangle$ of the cavity field, we can easily find that when the average photon number is zero, the concurrence reaches the maximal value

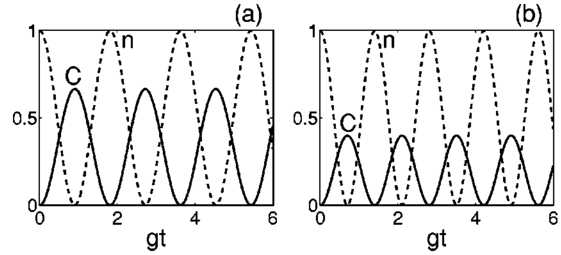

FIG. 1. Time evolutions of both the concurrence C (solid line) and the average photon number $n=\langle a^\dagger a\rangle$ of the cavity field (dashed line) plotted for (a) $N=3$ and (b) $N=5$.

of $2/N$ for any number N of the microcrystallites and vice versa. As an example for $N=3$ and 5, Fig. 1 clearly shows this point. Under the condition that all microcrystallites have the same coupling with the cavity field, when the concurrence reaches its maximum values, the state of the micro crystallites system is in the generalized W state, defined [20] to be $|W_N\rangle=(1/\sqrt{N})(|10\cdots 0\rangle+|010\cdots 0\rangle+\cdots+|0\cdots 01\rangle)$ and the cavity field is in the vacuum state, but in the anisotropic case, we cannot obtain the generalized W state for any condition. We know that a single exciton can be taken as a boson even for the quantum dots, so the assumption of the microcrystallites is not necessary in the case of the cavity field initially in the single-photon state. In the following two sections, we will mainly focus on the isotropic interaction of the cavity field and microcrystallites to discuss the entangled coherent excitonic states.

IV. PREPARATION OF THE ENTANGLED COHERENT EXCITONIC STATE

There are other two interesting orthogonal states called the even and odd coherent states (CS). These can be used as a robust qubit encoding for a single bosonic mode subject to amplitude damping, because the error caused by amplitude damping for this encoding can easily be corrected by a standard three-qubit error-correction circuit [21]. So, in this section, we will discuss how to realize symmetric sharing of entanglement between any pair of qubits defined by the even (odd) coherent excitonic states in semiconductor microcrystallites. It is well known that one can define the even CS as the zero-qubit state $|0\rangle$ and the odd CS as the one-qubit state $|1\rangle$ to encode a CNOT quantum gate [22], that is,

$$|0\rangle=N_+(|\alpha\rangle+|-\alpha\rangle), \tag{11a}$$

$$|1\rangle=N_-(|\alpha\rangle-|-\alpha\rangle), \tag{11b}$$

with the normalization constants $N_\pm=(2\pm 2e^{-2|\alpha|^2})^{-1/2}$ and $|\pm\alpha\rangle=\exp[-|\alpha|^2/2]\Sigma_{n=0}^{\infty}[(\pm\alpha)^n/\sqrt{n!}]|n\rangle$ are coherent states of a bosonic annihilation operator, e.g., the coherent states of the annihilation operator a for the cavity field. The even and odd coherent superpositions of the photon states in cavity quantum electrodynamics and those of motional states of trapped ions can be created by experimentalists [23] over the past several years. So, we can assume that the cavity field is initially either in the odd CS or in the even CS, and there are no excitons in any microcrystallite. In order to realize the symmetric sharing of entanglement, we assume that all microcrystallites have the same interaction with the cavity field, then the wave function of the whole system can be written by the factorization of the wave function [24] as follows

$$\begin{aligned}|\Psi(t)\rangle &= N_{\pm}U(t)[|\alpha\rangle_C \pm |-\alpha\rangle_C]|0\rangle^{\otimes N}\\ &= N_{\pm}(|\alpha u(t)\rangle_C|v(t)\alpha\rangle^{\otimes N}\\ &\quad \pm|-\alpha u(t)\rangle_C|-v(t)\alpha\rangle^{\otimes N}),\end{aligned} \tag{12}$$

with $u(t)=\cos(Gt)e^{-i\omega t}$ and $v(t)=-i[\sin(Gt)/\sqrt{N}]e^{-i\omega t}$, and the same coupling constants between microcrystallites and the cavity field are taken. We find that all excitonic coherent states $|v(t)\alpha\rangle$ in microcrystallites evolve periodically with time evolution, and their maximal amplitudes are $1/\sqrt{N}$ times the amplitude $|\alpha|$ of the coherent cavity field. We are interested in the pairwise entanglement in the system of N microcrystallites. After tracing out the cavity field and other degrees of freedom for $N-2$ microcrystallites, the reduced density operator for any pair can be expressed as

$$\begin{aligned}\rho(t) &= N_{\pm}^2\{(|v(t)\alpha\rangle\langle v(t)\alpha|)^{\otimes 2}+(|-v(t)\alpha\rangle\langle -v(t)\alpha|)^{\otimes 2}\\ &\quad \pm P(t)(|v(t)\alpha\rangle\langle -v(t)\alpha|)^{\otimes 2}\\ &\quad \pm P(t)(|-v(t)\alpha\rangle\langle v(t)\alpha|)^{\otimes 2}\},\end{aligned} \tag{13}$$

where $P(t)=\langle -u(t)\alpha|u(t)\alpha\rangle(\langle -v(t)\alpha|v(t)\alpha\rangle)^{N-2} = \exp[-2|\alpha|^2+4|\alpha|^2\sin^2(Gt)/N]$. We choose the time-dependent even and odd CS as the basis $\{|\tilde{0}\rangle,|\tilde{1}\rangle\}$ for each qubit in every microcrystallite as follows [25]:

$$|\tilde{0}\rangle = N_{+}(t)(|v(t)\alpha\rangle + |-v(t)\alpha\rangle), \tag{14a}$$

$$|\tilde{1}\rangle = N_{-}(t)(|v(t)\alpha\rangle - |-v(t)\alpha\rangle), \tag{14b}$$

where $N_{\pm}(t)$ are the normalization constants defined as $N_{\pm}(t)=(2\pm 2e^{-2[|\alpha|^2\sin^2(Gt)/N]})^{-1/2}$. Then the reduced density operator $\rho(t)$ can be given, in the basis $\{|\tilde{0}\tilde{0}\rangle,|\tilde{0}\tilde{1}\rangle,|\tilde{1}\tilde{0}\rangle,|\tilde{1}\tilde{1}\rangle\}$, in the following form:

$$\begin{aligned}\rho(t) &= \frac{N_{\pm}^2[1\pm P(t)]}{8N_{+}^4(t)}|\tilde{0}\tilde{0}\rangle\langle\tilde{0}\tilde{0}| + \frac{N_{\pm}^2[1\pm P(t)]}{8N_{-}^4(t)}|\tilde{1}\tilde{1}\rangle\langle\tilde{1}\tilde{1}|\\ &\quad + \frac{N_{\pm}^2[1\mp P(t)]}{8N_{+}^2(t)N_{-}^2(t)}\{|\tilde{0}\tilde{1}\rangle\langle\tilde{0}\tilde{1}| + |\tilde{0}\tilde{1}\rangle\langle\tilde{1}\tilde{0}| + |\tilde{1}\tilde{0}\rangle\langle\tilde{0}\tilde{1}|\\ &\quad + |\tilde{1}\tilde{0}\rangle\langle\tilde{1}\tilde{0}|\} + \frac{N_{\pm}^2[1\pm P(t)]}{8N_{+}^2(t)N_{-}^2(t)}\\ &\quad \times\{|\tilde{0}\tilde{0}\rangle\langle\tilde{1}\tilde{1}| + |\tilde{1}\tilde{1}\rangle\langle\tilde{0}\tilde{0}|\}.\end{aligned} \tag{15}$$

Following the same steps as for Eqs. (7) and (8), we obtain the concurrence corresponding to Eq. (15) as

$$C_{\pm}(t) = \frac{e^{(4/N)|\alpha|^2\sin^2(Gt)}-1}{e^{2|\alpha|^2}\pm 1}, \tag{16}$$

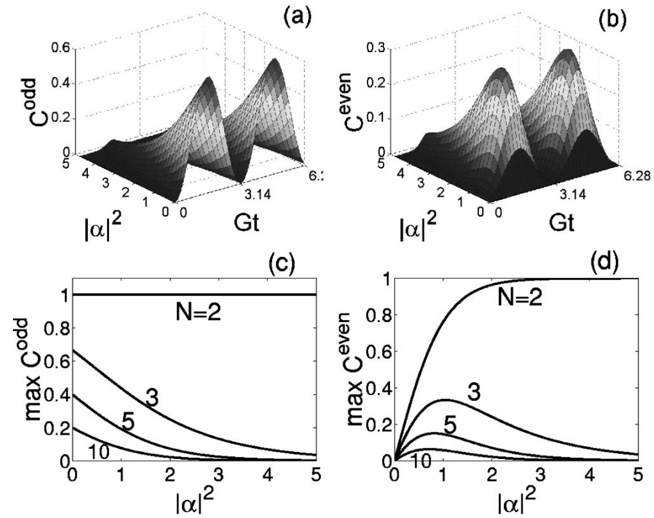

FIG. 2. The concurrences are plotted as a function of time and of intensity $|\alpha|^2$ of the cavity field when $N=3$ for the cavity field initially in (a) the odd CS and (b) the even CS. The maximal concurrences vs $|\alpha|^2$ for $N=2$, 3, 5, and 10 at optimum evolution times are plotted for the cavity field initially in (c) the odd CS and (d) the even CS.

where $+(-)$ means that the cavity field is initially in the even (odd) CS. It is also found that the concurrence periodically evolves. Although there is no simple analytical expression between the concurrence and the average photon number $\langle a^{\dagger}a\rangle$ of the cavity field as Eq. (10), we find that if the average photon number of the cavity field,

$$\langle a^{\dagger}a\rangle_{\pm} = |\alpha|^2\cos^2(Gt)\frac{1\mp e^{-2|\alpha|^2}}{1\pm e^{-2|\alpha|^2}}, \tag{17}$$

is zero, then the value of the concurrence reaches maximum and vice versa. So, for the fixed number N of the microcrystallites and the intensity $|\alpha|^2$ of the cavity field, the relationship of the time evolution between the concurrence and the average number of the cavity field is analogous to Fig. 1.

It is very clear that the values of the concurrence (16) depend on both the number N of the microcrystallites and the intensity $|\alpha|^2$ of the cavity field. The maximal concurrence with the cavity field initially in even CS or odd CS decreases with the increase of the microcrystallite's number N when the intensity $|\alpha|^2$ of the cavity field is fixed. It is because that $\exp\{(4/N)|\alpha|^2\sin^2(Gt)\}$ in Eq. (16) is a decreasing function of the number N, so a larger number N corresponds to a smaller concurrence. In the following, we have plotted Fig. 2 to show the time evolution of the concurrences and the relationship between the concurrence, and the intensity $|\alpha|^2$ for different microcrystallite number N. We find that the concurrence $C_{-}(t)\equiv C^{\mathrm{odd}}(t)$ periodically reaches its maximal value at the evolution times $Gt=(2n+1)(\pi/2)$ $(n=0,1,\ldots)$, and these points approach the upper bound value of $2/N$, which has been illustrated in Figs. 2(a) and 2(c) by an example for $N=3$, when $|\alpha|^2\to 0$. It is because the odd CS of the cavity field is reduced to the single-photon state when $|\alpha|^2\to 0$, so the concurrence $C_{-}(t)$ with initially the odd CS is reduced to Eq. (10) and approaches upper bound value of $2/N$. Figure 2(c) also shows the variation for the maximal value of the concurrence with the intensity $|\alpha|^2$ of the cavity field with different number N of the microcrystallites, we find that the maximal values of the concurrence decrease with the increase of the intensity $|\alpha|^2$, except that $N=2$. However, when $|\alpha|^2 \to 0$, the even CS of the cavity field is reduced to the vacuum state, the concurrence $C_+(t) \equiv C^{\text{even}}(t)$ with initially the even CS tends to zero. The maximal values of $C_+(t)$ increase with the increase of the intensity $|\alpha|^2$, but when the intensity $|\alpha|^2$ is greater than a threshold value, which is determined by

$$N=\frac{4|\alpha|^2\cosh|\alpha|^2}{|\alpha|^2e^{|\alpha|^2}+\cosh|\alpha|^2W\{-|\alpha|^2\operatorname{sech}|\alpha|^2e^{-|\alpha|^2\tanh|\alpha|^2}\}}, \tag{18}$$

for given number N $(N>2)$, where $W\{z\}$ is the product log function defined as the solution for w of $z=we^w$, then the concurrence gradually tends to zero. The maxima of $C_+(t)$ are reached at $Gt=(2n+1)(\pi/2)$ if N and $|\alpha|^2$ satisfy Eq. (18). We can also find when $|\alpha|^2$ is large enough, such that $|\alpha|^2 \approx 4$ when $N \geqslant 4$ and $|\alpha|^2 \approx 6$ when $N=3$ [see Figs. 2(a) and 2(b)], where bosonic approximation for excitons is still good, then $|\alpha v(t)\rangle$ is approximately orthogonal to $|-\alpha v(t)\rangle$ when $u(t)=0$, that is, $\langle -v(t)\alpha|v(t)\alpha\rangle \approx 0$. Under such condition, we can redefine two approximately orthogonal states $|1\rangle=|\alpha v(t)\rangle$ and $|0\rangle=|-\alpha v(t)\rangle$ as one-qubit state and zero-qubit state, then Eq. (15) for the reduced density operator of two subsystems of N $(N>2)$ microcrystallites can be simplified as

$$\rho \approx \frac{1}{2}[|00\rangle\langle 00| \pm |11\rangle\langle 11|], \tag{19}$$

which means that no entanglement appears for each pair of microcrystallites, then the concurrences are zero for cavity field initially in odd CS or even CS. So when the cavity field is initially in the even CS, the points of the maximal values for the concurrence must be between 0 and $|\alpha|^2$, with condition $\langle -v(t)\alpha|v(t)\alpha\rangle \approx 0$. These points are determined by Eq. (18).

The optimum values of $|\alpha|^2$ and N maximizing $C_+(t)$ obtained from Eq. (18) and checked directly by numerical maximization of Eq. (16) are, e.g., as follows: $|\alpha|^2 = 3/2\ln(2) \approx 1.04$ for $N=3$, and $|\alpha|^2=\ln(1+\sqrt{2}) \approx 0.88$ for $N=4$. While for $N=5$ the maximum is at $|\alpha|^2=\ln[16/9 + 10^{1/3}20/27 + 10^{2/3}10/27]/2 \approx 0.81$. For $N=6$ and 8, analytical expressions can also be found. However, for other cases (i.e., $N=7$ and $N \geqslant 9$) there are no compact-form analytical formulas for $|\alpha|^2$ corresponding to the maximal concurrence. The above numerical calculations can be seen from Figs. 2(b) and 2(d). Figure 2 also shown when the number N of the microcrystallites is equal to 2, the maximally entangled states can be prepared for any intensity of the cavity field with initially the odd CS, but for the cavity field with initially the even CS, we can approximately prepare a maximally entangled state when the average photon number $|\alpha|^2$ is slightly larger than 1, e.g., $|\alpha|^2 \geqslant 3$ [see Fig. 2(d)].

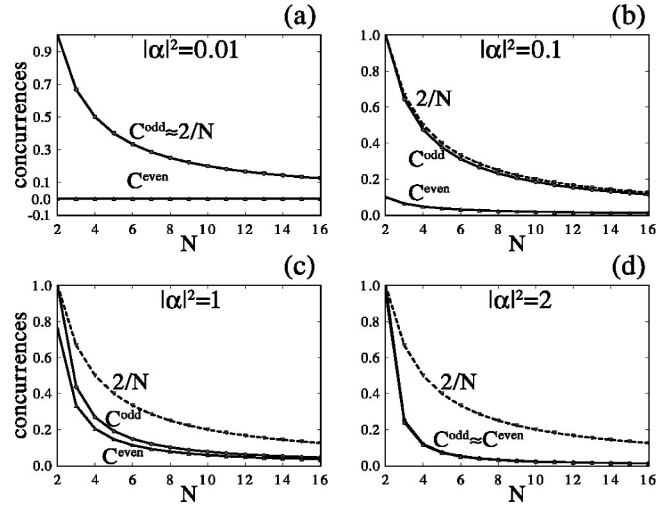

FIG. 3. The maximum values of the concurrences are depicted as a function of the number N of semiconductor microcrystallites for the different intensities of the cavity field with initially the odd or even CS for (a) $|\alpha|^2=0.01$, (b) $|\alpha|^2=0.1$, (c) $|\alpha|^2=1$, and (d) $|\alpha|^2=2$, and dashed curve corresponds to the maximum possible concurrence, given by $2/N$, between any pair of qubits.

As a comparison, we numerically show that the maximal values of the concurrence in the case of the cavity field initially in the odd or even CS for the different number N of the microcrystallites and the different intensity $|\alpha|^2$ of the cavity field from Figs. 3(a) to 3(d). Figure 3 shows that the maximal values of the concurrences with initially the odd or even CS of the cavity field approach each other with increasing the intensity of the cavity field, but they are lower than the upper bound values of $2/N$ for the concurrence. From Fig. 3(a), we can also find that the concurrence with the cavity field initially in the odd or even CS gradually approaches its maximal possible values of $2/N$ or zero when the intensity of the cavity field tends to zero, that is, $|\alpha|^2 \to 0$.

V. DECAY OF THE ENTANGLED EXCITON STATES

The quantum computation and the quantum information take their power from the superpositions and entanglement of the quantum states, however, the necessary coupling of the system to the environment tends to destroy this coherence and reduces the degree of the entanglement with the time evolution of the total system. So, in this section, we will discuss the decay of the entangled exciton states. For simplification of our discussion, we will model the interaction between the system and environment as follows. We assume that there is no interaction between the cavity field and environment. The dissipation of the system energy comes from the interaction of the excitons in the microcrystallites with environment, here modeled as thermal radiation fields at zero temperature. We limit our discussion to the completely identical N microcrystallites and the coupling constants between microcrystallites, and cavity field are the same. Under the above assumptions, the Hamiltonian for the system, environments, and their interactions can be written as

$$H=\hbar\omega a^{\dagger}a+\hbar\omega\sum_{j=1}^{N} b_j^{\dagger}b_j+\hbar g\sum_{j=1}^{N} (a^{\dagger}b_j+ab_j^{\dagger})+\hbar\sum_{j=1}^{N}\sum_{k} \omega_{j,k}a_{j,k}^{\dagger}a_{j,k}+\hbar\sum_{j=1}^{N}\sum_{k} g_{j,k}(a_{j,k}^{\dagger}b_j+a_{j,k}b_j^{\dagger}), \tag{20}$$

where $a_{j,k}(a_{j,k}^{\dagger})$ are annihilation (creation) operators of radiation fields with frequency $\omega_{j,k}$ and $g_{j,k}$ are coupling constants between the jth microcrystallite and radiation fields. For simplicity, we assume that all $g_{j,k}$ are independent of the microcrystallite size. We assume that each microcrystallite separately interacts with the environment, but the dissipative dynamics is same for all the microcrystallites. The latter assumption is not necessary but only simplifies the degree of algebra complexity for calculation.

We can obtain the Heisenberg equations of motion for each operator as follows:

$$\frac{\partial B_j}{\partial t}=-igA-i\sum_{k} g_{j,k}A_{j,k}e^{-i(\omega_{j,k}-\omega)t}, \tag{21a}$$

$$\frac{\partial A}{\partial t}=-ig\sum_{j=1}^{N} B_j, \tag{21b}$$

$$\frac{\partial A_{j,k}}{\partial t}=-ig_{j,k}B_je^{i(\omega-\omega_{j,k})t}, \tag{21c}$$

where the transformations $a(t)=A(t)e^{-i\omega t}$, $b_j(t)=B_j(t)e^{-i\omega t}$, and $a_{j,k}(t)=A_{j,k}(t)e^{-i\omega_{j,k}t}$ are made. From Eq. (21c), we have

$$A_{j,k}(t)=A_{j,k}(0)-ig_{j,k}\int_0^t \mathrm{d}t' B_j(t')e^{i(\omega-\omega_{j,k})t'}. \tag{22}$$

We replace $A_{j,k}(t)$ in Eqs. (21a) by Eq. (22), then obtain the new equation as

$$\frac{\partial B_j}{\partial t}=-igA-i\sum_{k} g_{j,k}A_{j,k}(0)e^{-i(\omega_{j,k}-\omega)t}-\sum_{k}|g_{j,k}|^2\int_0^t dt' B_j(t')e^{-i(\omega_{j,k}-\omega)(t-t')}. \tag{23}$$

We can apply the Laplace transform and the Wigner-Weisskopf approximation [26] to Eqs. (21b) and (23), so that we have the solution of the cavity field as

$$A(t)=u'(t)a(0)-i\sum_{j} v'(t)b_j(0)+\sum_{j}\sum_{k} v_{j,k}(t)a_{j,k}(0), \tag{24}$$

where conditions $A(0)=a(0)$, $B_j(0)=b_j(0)$, and $A_{j,k}(0)=a_{j,k}(0)$ were used, and

$$u'(t)=e^{-(\gamma/4)t}\left(\frac{\gamma}{4\delta}\sin(\delta t)+\cos(\delta t)\right), \tag{25a}$$

$$v'(t)=\frac{g}{\delta}e^{-(\gamma/4)t}\sin(\delta t), \tag{25b}$$

$$\delta=\sqrt{Ng^2-(\gamma/4)^2}, \tag{25c}$$

where the small Lamb frequency shift is neglected and the decay rate $\gamma=2\pi\rho(\omega_0)|g(\omega_0)|^2$. We also use the former assumptions under which all microcrystallites are the same and have the same dissipative dynamics so that the decay rates γ of each microcrystallite and the functions $v'(t)$ of each term including operators $b_j(0)$ are the same. When the environment is considered, the qubits for each microcrystallite should be redefined as

$$|\tilde{0}\rangle=M_+(t)[|v'(t)\alpha\rangle+|-v'(t)\alpha\rangle], \tag{26a}$$

$$|\tilde{1}\rangle=M_-(t)[|v'(t)\alpha\rangle-|-v'(t)\alpha\rangle], \tag{26b}$$

with normalization constant $M_\pm(t)=[2\pm 2e^{-2|v'(t)\alpha|^2}]^{-1/2}$. Now, we will investigate the decay when the cavity field is initially in the even and odd CS, but no excitons are initially in any microcrystallite. After tracing out the degrees of the environments and other $N-2$ microcrystallites for the time-dependent wave function of the whole system, which can also be obtained using the factorized form of the wave function, we get the reduced density operator for any two qubits as

$$\rho'_\pm(t)=\frac{N_\pm^2[1\pm P'(t)]}{8M_+^4(t)}|\tilde{0}\tilde{0}\rangle\langle\tilde{0}\tilde{0}|+\frac{N_\pm^2[1\pm P'(t)]}{8M_-^4(t)}|\tilde{1}\tilde{1}\rangle\langle\tilde{1}\tilde{1}|+\frac{N_\pm^2[1\mp P'(t)]}{8M_+^2(t)M_-^2(t)}\{|\tilde{0}\tilde{1}\rangle\langle\tilde{0}\tilde{1}|+|\tilde{0}\tilde{1}\rangle\langle\tilde{1}\tilde{0}|+|\tilde{1}\tilde{0}\rangle\langle\tilde{0}\tilde{1}|+|\tilde{1}\tilde{0}\rangle\langle\tilde{1}\tilde{0}|\}+\frac{N_\pm^2[1\pm P'(t)]}{8M_+^2(t)M_-^2(t)}\times\{|\tilde{0}\tilde{0}\rangle\langle\tilde{1}\tilde{1}|+|\tilde{1}\tilde{1}\rangle\langle\tilde{0}\tilde{0}|\}, \tag{27}$$

with $P'(t)=\exp[-2|\alpha|^2(1-2|v'(t)|^2)]$. Then the concurrences $C'_\pm(t)$ corresponding to Eq. (27) can be obtained as

$$C'_\pm(t)=\frac{e^{4|\alpha v'(t)|^2}-1}{e^{2|\alpha|^2}\pm 1}, \tag{28}$$

where $v'(t)$ is determined by Eq. (25b). We find that the concurrence (16) is modified and becomes of the form (28) after the effect of the environment is taken into account. Now we will make further approximation. We assume that the couplings of the cavity field with microcrystallites are stronger than the decay of the exciton, i.e., $g\gg\gamma$. In this case we can approximately obtain the concurrence $C'_\pm(t)$ as

$$C'_\pm(t)\approx\frac{\exp[(4|\alpha|^2/N)\sin^2(gt\sqrt{N})e^{-(\gamma/2)t}]-1}{e^{2|\alpha|^2}\pm 1}. \tag{29}$$

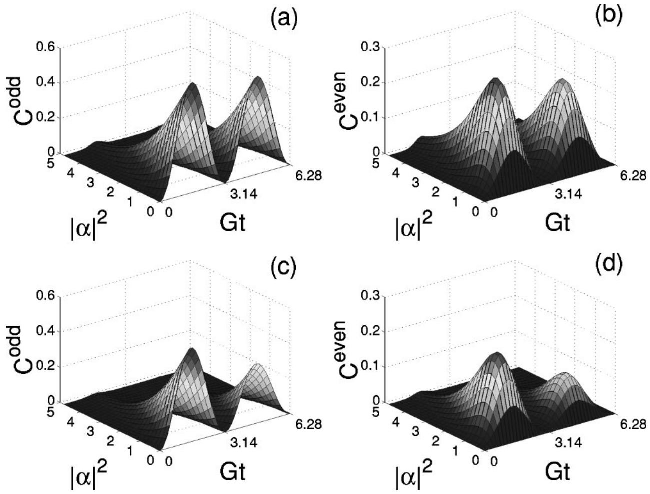

FIG. 4. The time evolution of the concurrences depicted with the number $N=3$ of semiconductor microcrystallites for $\gamma/g=0.13$ (a), (b) and $\gamma/g=0.5$ (c), (d) when the cavity field is initially in the odd CS (a), (c) or in the even CS (b), (d).

Equation (29) shows that the entanglement between any pair of qubits decays in an oscillating form. It also shows that the increase of the microcrystallite number N results in the decrease of the concurrence when the decay rate γ and the intensity $|\alpha|^2$ are fixed. For convenience of discussion, we can rescale time in Eq. (29) as $t'=Gt$, then we can find that if the coherent intensity $|\alpha|^2$ of the cavity field and the number N of microcrystallites are given, then the larger ratio between decay rate γ and the coupling constant g corresponds to the faster reduction of the concurrence of the entangled qubits. But if the ratio of γ and g is given and the number N is fixed, then the higher intensity of the cavity field corresponds to the smaller concurrence. As an example, Fig. 4 plots the variation of concurrence with $N=3$ for a reasonably good cavity $\gamma/g=0.13$ [in Figs. 4(a) and 4(b)] [9], or for a bad cavity $\gamma/g=0.5$ [in Figs. 4(c) and 4(d)] according to Eq. (28). Figure 4 clearly demonstrates our above discussions.

VI. CONCLUSIONS

We have studied an excitonic-state implementation of the multiparticle entanglement based on N spatially separated semiconductor microcrystallites. The interaction among the microcrystallites is mediated by a single-mode cavity field. We find that the entanglement (measured by the concurrence) between any pair of qubits that are defined by the excitonic number states (vacuum and a single-exciton states) or the coherent excitonic states (odd and even CS), depends on the interaction between the cavity field and the semiconductor microcrystallites. The entanglement between any pairs is different from one another for the anisotropic case. When all microcrystallites have the same interaction with the cavity field, the maximal degree of the entanglement between any pair of qubits is the same. This condition can, probably, be satisfied with the development of the fabrication techniques for quantum dots and the semiconductor microcavity quantum electrodynamics. So, the symmetric sharing of the entanglement between any pair of N qubits in such a system is realizable only when the interaction between N spatially separated semiconductor microcrystallites and the cavity field is isotropic. Under the isotropic-interaction condition, when the excitonic system reaches maximal entanglement, all photons in the cavity are transformed into the excitons in the system of the semiconductor microcrystallites. The generalized W state and the maximal degree $2/N$ of entanglement can be obtained for the cavity field initially in the single-photon state. But if the cavity field is initially in the odd or even CS, we cannot obtain the maximal degree of entanglement $2/N$, except the special case where the cavity field is initially in the odd CS and there are two microcrystallites in the cavity [27].

We have also investigated the decay of any pair of the entangled qubits defined by the odd and even CS. The entanglement between any pair of qubits decreases because of the dissipation of the system energy to the environment. If the coherent intensity of the cavity field and the number of microcrystallites are given, then with the rescaled time, the larger ratio between the decay rate γ and the coupling constant g corresponds to the faster reduction of the concurrence of the entangled qubits. For the given ratio between the decay rate γ and the coupling constant g, and the coherent intensity of the cavity field, the increase of the microcrystallite number N results in the decrease of the concurrence. But if the ratio γ/g is given, and the number N is fixed, then the higher intensity of the cavity field corresponds to the smaller concurrence. Practically, the quality of the entanglement can be improved with the appearance of the new processing techniques and the ultrahigh finesse cavities [9]. Finally, we should point out that our discussion is limited to the preparation of the entangled coherent excitonic states, but we cannot control them using this model.

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[1] D. Bouwmeester, J.-W. Pan, M. Daniel, H. Weinfurter, and A. Zeilinger, Nature (London) **390**, 575 (1997).
[2] A. Furusawa, J.I. Sørensen, S.L. Braunstein, C.A. Fuchs, H.J. Kimble, and E.S. Polzik, Science **282**, 706 (1998).
[3] D. M. Greenberger, M. Horne, and A. Zeilinger, *Bell's Theorem, Quantum Theory, and Conceptions of the Universe*, edited by M. Kafatos (Kluwer, Dordrecht, 1989).
[4] W.K. Wootters, e-print quant-ph/0001114.
[5] M. Koashi, V. Bužek, and N. Imoto, Phys. Rev. A **62**, 050302 (2000).

[6] W. Dür, Phys. Rev. A **63**, 020303 (2001).

[7] K.M. O'Connor, and W.K. Wootters, Phys. Rev. A **63**, 052302 (2001).

[8] D. Loss and D.P. DiVincenzo, Phys. Rev. A **57**, 120 (1998); Xuedong Hu and S. Das Sarma, *ibid.* **61**, 062301 (2000); T. Ohshima, *ibid.* **62**, 062316 (2000).

[9] A. Imamoḡlu, D.D. Awschalom, G. Burkard, D.P. DiVincenzo, D. Loss, M. Sherwin, and A. Small, Phys. Rev. Lett. **83**, 4204 (1999); S.N. Molotkov, JETP Lett. **64**, 237 (1996); T. Tanamoto, Phys. Rev. A **61**, 022305 (2000); G. Burkard, D. Loss, and D.P. DiVincenzo, Phys. Rev. B **59**, 2070 (1999); A. Miranowicz, Ş.K. Özdemir, Yu-xi Liu, M. Koashi, N. Imoto, and Y. Hirayama, Phys. Rev. A **65**, 062321 (2002).

[10] K. Obermayer, G. Mahler, and H. Haken, Phys. Rev. Lett. **58**, 1792 (1987); E. Biolatti, Rita C. Iotti, P. Zanardi, and F. Rossi, *ibid.* **85**, 5647 (2000).

[11] S.N. Molotkov and S.S. Nazin, JETP Lett. **63**, 687 (1996).

[12] E. Dekel, D. Gershoni, E. Ehrenfreund, J.M. Garcia, and P.M. Petroff, Phys. Rev. B **61**, 11 009 (2000); P. Chen, C. Piermarocchi, and L.J. Sham, Phys. Rev. Lett. **87**, 067401 (2001).

[13] G. Chen, N.H. Bonadeo, D.G. Steel, D. Gammon, D.S. Katzer, D. Park, and L.J. Sham, Science **289**, 1906 (2000); M. Bayer, P. Hawrylak, K. Hinzer, S. Fafard, M. Korkusinski, Z.R. Wasilewski, O. Stern, and A. Forchel, *ibid.* **291**, 451 (2001); L. Quiroga and N.F. Johnson, Phys. Rev. Lett. **83**, 2270 (1999).

[14] T.H. Stievater, Xiaoqin Li, D.G. Steel, D. Gammon, D.S. Katzer, D. Park, C. Piermarocchi, and L.J. Sham, Phys. Rev. Lett. **87**, 133603 (2001); H. Kamada, H. Gotoh, J. Temmyo, T. Takagahara, and H. Ando, *ibid.* **87**, 246401 (2001).

[15] I.L. Chuang, D.W. Leung, and Y. Yamamoto, Phys. Rev. A **56**, 1114 (1997).

[16] E. Hanamura, Phys. Rev. B **38**, 1228 (1988).

[17] L. Belleguie and L. Bányai, Phys. Rev. B **44**, 8785 (1991); L. Bányai and S.W. Koch, *Semiconductor Quantum Dots* (World Scientific, Singapore, 1993).

[18] A. Engelmann, V.I. Yudson, and P. Reineker, Phys. Rev. B **57**, 1784 (1998).

[19] S. Hill and W.K. Wootters, Phys. Rev. Lett. **78**, 5022 (1997); W.K. Wootters, *ibid.* **80**, 2245 (1998).

[20] W. Dür, G. Vidal, and J.I. Cirac, Phys. Rev. A **62**, 062314 (2000); Xiaogunag Wang, *ibid.* **64**, 012313 (2001).

[21] P.T. Cochrane, G.J. Milburn, and W.J. Munro, Phys. Rev. A **59**, 2631 (1999).

[22] M.C. de Oliveira and W.J. Munro, Phys. Rev. A **61**, 042309 (2000); M.C. de Oliveira, M.H.Y. Moussa, and S.S. Mizrahi, *ibid.* **61**, 063809 (2000).

[23] M. Brune, E. Hagley, J. Dreyer, X. Maître, A. Maali, C. Wunderlich, J.M. Raimond, and S. Haroche, Phys. Rev. Lett. **77**, 4887 (1996); C. Monroe, D.M. Meekhof, B.E. King, and D.J. Wineland, Science **272**, 1131 (1996).

[24] C.P. Sun, Y.B. Gao, H.F. Dong, and S.R. Zhao, Phys. Rev. E **57**, 3900 (1998).

[25] H. Jeong, M.S. Kim, and J. Lee, Phys. Rev. A **64**, 052308 (2001).

[26] W.H. Louisell, *Quantum Statistical Properties of Radiation* (Wiley, New York, 1973).

[27] Yu-xi Liu, Ş.K. Özdemir, M. Koashi, and N. Imoto, Phys. Rev. A **65**, 042326 (2002).