An Introduction to Quantum Teleportation

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Recent experiments confirm that quantum teleportation is possible at least for states of photons and nuclear spins. The quantum teleportation is not only a curious effect but a fundamental protocol of quantum communication and quantum computing. The principles of the quantum teleportation and the entanglement swapping are explained, and physical realizations of teleportation of optical and atomic states are discussed.

I. INTRODUCTION

Teleportation is commonly understood as a fictional method for disembodied transport: An object or person is disintegrated at one place and it is perfectly reconstructed somewhere else. Thus, teleportation can be compared to transmission of a three-dimensional object using a kind of super fax machine which, however, destroys the original object on scanning. In a sense, the dokodemo-door (literally: door-to-anywhere) from the Japanese Manga story "Dora-e-mon" can be considered as a teleporting device (teleporter). In the movie "Star Trek", teleportation serves as a standard transportation mean almost as common as elevators. A Hollywood vision of possible dangers of non-perfect teleportation has been created in the famous sci-fi movies "The Fly" (1958,1986) and their sequels. Obviously, all these examples sound completely unrealistic.

Until recently, physicists had ruled out teleportation because of the implication of Heisenberg's uncertainty principle formulated as the No Cloning Theorem, which prohibits making an exact copy of an unknown quantum state. Yet, in 1993, an international team of six scientists including Charles Bennett [1] demonstrated that it is possible to transmit an unknown quantum state from one place to another without propagation of the associated physical object through the intervening space by way of a process called the *quantum teleportation* (QT). The success of the first experimental teleportation realized by Anton Zeilinger's group of the University of Insbruck in 1997 [2] was the cover story of many journals, including "Scientific American" [3], and even newspapers all around the world. These spectacular scientific achievements have caused much rumour.

Obviously, the underlying principles of the quantum teleportation are fundamentally different from those of the dokodemo-door or the "Star Trek" teleporters of beaming people around. The phenomenon that makes quantum teleportation possible is the *quantum entanglement* also referred to as the Einstein-Podolsky-Rosen (EPR) correlations [4]. Entanglement is a special interrelationship between objects, in which measuring one object instantly influences the other, even if the two are completely isolated and separated from one another. For example, if two photons come into contact with each other, they can become entangled: The polarization of each photon is in a fuzzy, undetermined state, yet the two photons have a precisely defined interrelationship. If one photon is later measured by linear polarizer to have, say, a vertical polarization, then the other photon must collapse into the complementary state of horizontal polarization. But if the entangled photon was measured by circular polarizer, instead of the linear one, to have, say, a right-circular polarization then the other photon had to collapse into the complementary state of the left-circular polarization. Thus, if one of the entangled photons is measured in any basis to have a definite polarization, then the state of the other must be exactly complementary to this polarization. It is so bizarre that even Albert Einstein, who predicted this effect, considered it not to be real and called it spooky [4]. An entangled state of a system consisting of two subsystems cannot be described as a product of the quantum states of these subsystems. In this sense, the entangled system is considered inseparable and nonlocal. Entanglement is usually manifested in systems consisting of a small number of microscopic particles but, recently, it has also been experimentally observed in macroscopic systems of 10^{12} atoms [5], which sounds promising for further research in teleportation of states of mesoscopic or even macroscopic objects. Entanglement is one of the most profound features of quantum mechanics having fundamental importance not only for quantum teleportation but also for quantum computing and quantum cryptography.

II. PRINCIPLES OF QUANTUM TELEPORTATION

Here, we present the quantum teleportation protocol devised by Bennett et al. [1] in 1993. This protocol is limited to teleportation of states of a two-level quantum system, referred to as the quantum bit or *qubit.* Nevertheless, generalization for teleportation of states of multi-level or infinitely-level systems is conceptually simple.

Qubit states to be teleported can be chosen as, for example, the polarization states of single photon ($|\updownarrow\rangle$ and $|\leftrightarrow\rangle$, or $|+\rangle$ and $|-\rangle$), the photon-number states of a

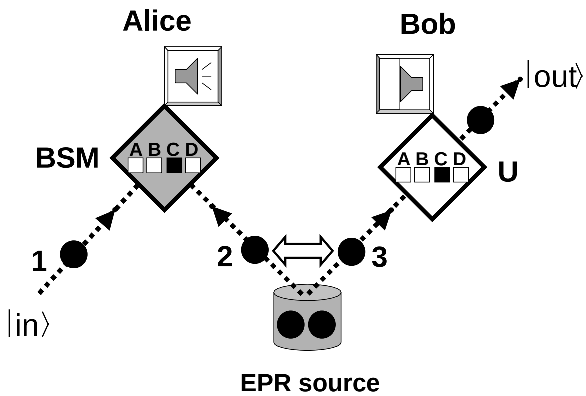

FIG. 1: Principles of quantum teleportation in the original scheme of Bennett et al. [1]: Alice has a qubit 1 in an initial state $|in\rangle$, which she wants to teleport to Bob. Besides Alice and Bob share an ancillary entangled (marked by white arrow) pair of qubits (2 and 3) emitted by an Einstein-Podolsky-Rosen (EPR) source. Alice performs a joint Bell state measurement (BSM) on her qubit 1 and one of the ancillaries (say qubit 2), projecting them onto one (say C as marked by black box) of four orthogonal entangled states called the Bell states (A, B, C, and D). Alice then sends to Bob the result of her measurement by classical communication (symbolized here by speakers). Dependent on this information, Bob does not change the state of his qubit 3 (case A) or performs a unitary transformation (U) on it (cases B-D) resulting in obtaining the output state $|out\rangle$ of qubit 3 being exactly the same as the input state $|in\rangle$ of the original qubit 1.

cavity ($|0\rangle$ and $|1\rangle$), the spin states of a spin-$\frac{1}{2}$ particle (like electron) ($|\uparrow\rangle$ and $|\downarrow\rangle$), ground and excited states of an atom or ion ($|g\rangle$ and $|e\rangle$), or others. But it should be stressed that all these realizations of qubits are mathematically equivalent. Thus, to describe the principles of QT in this section, we use the standard information notation of $|0\rangle$ and $|1\rangle$ for qubit states.

The heart of teleportation are entangled qubits and the measurement of their joint state performed in the basis of four maximally entangled states, which are referred to as the *Bell states* or the EPR states:

$$\begin{aligned}|\Phi_A\rangle &= \tfrac{1}{\sqrt{2}}(|0\rangle|1\rangle - |1\rangle|0\rangle)\\ |\Phi_B\rangle &= \tfrac{1}{\sqrt{2}}(|0\rangle|1\rangle + |1\rangle|0\rangle)\\ |\Phi_C\rangle &= \tfrac{1}{\sqrt{2}}(|0\rangle|0\rangle - |1\rangle|1\rangle)\\ |\Phi_D\rangle &= \tfrac{1}{\sqrt{2}}(|0\rangle|0\rangle + |1\rangle|1\rangle) \qquad (1)\end{aligned}$$

Quantum teleportation is a scheme by which the state of a qubit can be transmitted from one place to another by classical communication, provided that the sender (say Alice) and the receiver (say Bob) have previously shared halves of two-qubit entangled state. In detail, this is how it works: Assume that Alice has a qubit 1 in the state $|in\rangle_1 = a|0\rangle_1 + b|1\rangle_1$ with unknown amplitudes a and b normalized to unity, $|a|^2 + |b|^2 = 1$. In addition, Alice has qubit 2 initially entangled to Bob's qubit 3 being in one of the Bell states, e.g., $|\Phi_A\rangle_{23}$, where subscripts 2 and 3 refer to relevant qubits. The goal of teleportation is to transmit the state of Alice's qubit 1 to Bob's qubit 3. The total initial state, given by $|in\rangle_1|\Phi_A\rangle_{23}$, can be rewritten in the Bell basis as

$$\begin{aligned}&\tfrac{1}{2}[|\Phi_A\rangle_{12}(a|0\rangle_3 + b|1\rangle_3) + |\Phi_B\rangle_{12}(a|0\rangle_3 - b|1\rangle_3)\\ &-|\Phi_C\rangle_{12}(a|1\rangle_3 + b|0\rangle_3) + |\Phi_D\rangle_{12}(a|1\rangle_3 - b|0\rangle_3)] \quad (2)\end{aligned}$$

simply by regrouping terms and omitting unimportant global phase factor ($e^{i\pi} = -1$). Alice measures the joint state of qubits 1 and 2 in the Bell basis obtaining one of four possible results {A,B,C,D}≡{00,01,10,11} (in classical bit notation). The box BSM in figure 1 represents this Bell state measurement. No matter what the two-qubit input state is, Alice's measurement gives a uniformly distributed random two-bit classical result. However, this measurement clarifies the difference between Alice's initial qubit state and Bob's qubit: (A) If the measured qubits 1 and 2 are found to be in state $|\Phi_A\rangle_{12}$, then Alice's classical output is 00 and Bob's state is $|out\rangle_3 = a|0\rangle_3 + b|1\rangle_3$, which is exactly the initial Alice's state $|in\rangle_1$ without need to apply any additional transformation ($U_A = I$); (B) If qubits 1 and 2 are found to be in state $|\Phi_B\rangle_{12}$, then Alice's output is 01 and Bob's state is $|out_B\rangle_3 = a|0\rangle_3 - b|1\rangle_3$, which differs from $|in\rangle_1$. However, a little thought shows that by applying a simple phase flip $|x\rangle \mapsto (-1)^x|x\rangle$ ($x = 0, 1$), which is realizable by the Pauli operator $U_B = \sigma_z$, Bob gets state $|out\rangle_3 = U_B|out_B\rangle_3$ being the exact copy of Alice's state $|in\rangle_1$. Similarly, in cases C and D, Bob applies the proper unitary transformations $U_C = -\sigma_x$ (a bit flip $|x\rangle \mapsto -|x \oplus 1\rangle$) and $U_D = -i\sigma_y$ (a bit flip + phase flip $|x\rangle \mapsto (-1)^{x+1}|x \oplus 1\rangle$) to obtain exactly the same state as Alice's state $|in\rangle_1$. Thus, we conclude that to make a successful teleportation, Alice has to inform Bob which of the four states she measured, i.e., she must send him two bits of classical information. Only then Bob can perform the correction procedure by applying the appropriate unitary transformation U (I, σ_z, $-\sigma_x$ or $-i\sigma_y$) to recover the initial Alice state. It is worth stressing that the transmission of qubit states cannot be accomplished faster than light because Bob must wait for Alice's measurement result to arrive before he can recover the quantum state. Another important point is that Alice by performing the Bell state measurement destroys the initial state $|in\rangle_1$ of her photon. This loss of Alice's state is the reason that QT does not violate the no-cloning principle.

So far, we have explained the teleportation for qubit states only. As an example, we will discuss another protocol, which can be used for teleportation of qubit states but also for state truncation. In figure 2, we present a device proposed by David Pegg of Griffith University, and Lee Phillips and Stephen M. Barnett of the University of Strathclyde [6], which is referred to as the *quantum scissors*. In fact, this process can be considered as the quantum teleportation since it is based on the same principles as the original Bennett's scheme: (i) entanglement,

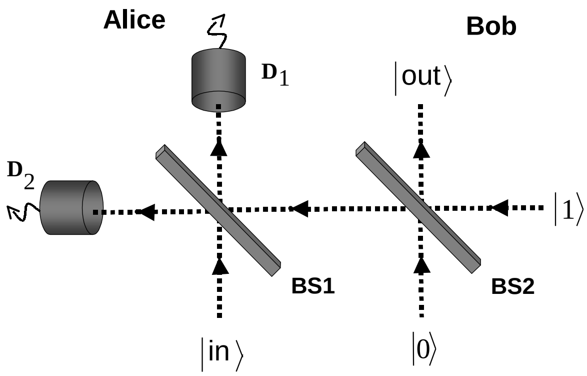

FIG. 2: Quantum teleportation and state truncation using quantum scissors [6]. If the inputs to the beam splitter BS2 are single-photon state $|1\rangle$ and vacuum $|0\rangle$, and Alice records one count at detector D_1 and none at D_2, then the input state $|in\rangle = c_0|0\rangle + c_1|1\rangle + c_2|2\rangle + \cdots$ is teleported and truncated to $|out\rangle = c_0|0\rangle + c_1|1\rangle$ (with proper normalization). Thus, if the input is the qubit state $|in\rangle = c_0|0\rangle + c_1|1\rangle$ then it is just teleported (without truncation). Bob, if informed by Alice about her measurement result, knows that $|out\rangle = |in\rangle$. But it is equally likely that Alice records one count at D_2 and none at D_1. In this case, $|out\rangle$ would be phase flipped in comparison to $|in\rangle$. Thus Bob, after receiving the information from Alice about her measurement output, must perform on his state $|out\rangle$ a simple unitary transformation of $|x\rangle \mapsto (-1)^x|x\rangle$ to get the exact of copy of Alice's state. If Alice records zero or two counts, then the teleportation protocol fails. It follows that the probability of successful teleportation is one in two.

and so nonlocality, and (ii) the Bell-state measurement (the projection postulate). The entangled state is generated by Bob's beam splitter BS2 from the input states $|0\rangle$ and $|1\rangle$. Please note that no light from input qubit $|in\rangle$ reaches output qubit $|out\rangle$ so, indeed, the process is a nonlocal phenomenon relying on quantum entanglement. Alice implements her Bell-state measurement by beam splitter BS1 and photon counters D_1 and D_2. And, as required for teleportation, the original Alice's state $|in\rangle$ is destroyed by her measurement. The first experiment with the quantum scissors has been done by Alex Lvovsky's group of the University of Konstanz [7].

Bennett's protocol is deterministic (unconditional), which means that every qubit state entering the setup can be teleported for any output of Alice's measurement. The experimental protocols of Akira Furusawa et al. carried out at the California Institute of Technology [8] and Michael Nielsen et al. of the Los Alamos National Laboratory [9] are also deterministic. However, the experimental protocols of Zeilinger's group [2] and Francesco De Martini's group of La Sapienza University in Rome [10], or also the quantum scissors are only probabilistic (conditional): The teleportation is successful conditional on appropriate results of Alice's measurement.

III. ATOMIC-STATE TELEPORTATION

In the former section, we have described schemes for quantum teleportation of optical qubit states since the first proposal [1] and the majority of experimental implementations of teleportation to date [2, 8, 10] were performed in optical domain. However, from the practical point of view, photonic qubits are not ideal for the long-term storage of quantum information since they are very difficult to keep in certain place as, e.g., light trapped in a cavity eventually leaks out. Thus, in the quest for practical applications of quantum computers, the teleportation of states of nuclear or atomic qubits attracts an increasing interest to mention the spectacular experiment of Nielsen et al. [9] of complete quantum teleportation of states of nuclear qubits, where quantum state of carbon nucleus was teleported to a hydrogen nucleus over interatomic distances using nuclear magnetic resonance. Atoms (their electrons or nuclei) are ideal for the long-term storage of quantum information. Unfortunately, atoms move slowly and also interact strongly with their environment and therefore they are not ideal for the quantum information transfer at long distances. By contrast, photonic states are ideal for the information transmission but not the information storage. Here, we will describe another interesting proposal of Peter Knight's group of Imperial College [11] for teleportation of atomic states over macroscopic distances. This scheme takes advantages both of photons for transfer and atoms for storage of quantum information.

The crucial role in this teleportation protocol plays cavity spontaneous photon leakage. It is a well accepted fact that spontaneous decay of excited quantum systems is a mechanism of their coherence loss (referred to as the decoherence) and therefore usually plays a destructive role in quantum information processing. However, Knight et al. have shown how detection of decay can be used constructively not only for establishment of entanglement but also for the complete quantum information processing such as teleportation. This surprising result can be understood by recalling the fact that a detected decay is a measurement on the state of the system from which the decay ensues.

Outline of the scheme is shown in figure 3. The setup consists of two optical cavities: Alice's C_a and Bob's C_b tuned to the same frequency $\omega_a^{(C)} = \omega_b^{(C)}$. Each cavity contains a single trapped three-level atom (A_a or A_b), which is illuminated within a proper period of time by classical laser field (L_a or L_b). The atomic energy levels are depicted in figure 4. By illuminating the atoms with the classical laser field of frequency $\omega_n^{(L)}$, Alice (designated by subscript $n = a$) and Bob ($n = b$) can drive the transition $|e\rangle_n \leftrightarrow |r\rangle_n$. The other transition of $|g\rangle_n \leftrightarrow |r\rangle_n$ is driven by the quantized cavity field of frequency $\omega_n^{(C)}$. It is important to assume that detunings Δ_n are large enough such the upper levels $|r\rangle_n$ can effectively be decoupled (so neglected) from

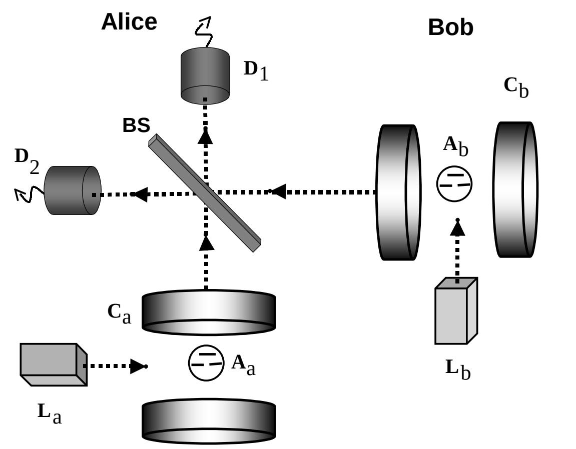

FIG. 3: Atomic-state teleportation via decay [11]. The unknown state of Alice's atom A_a trapped in her cavity C_a can be teleported to Bob's atom A_b trapped in a distant cavity C_b by the joint detection by D_1 and D_2 of photons leaking from the cavities. At the preparation stage, the atoms should be illuminated by lasers L_a and L_b. The state to be teleported is the internal state of an atom being ideal for storing quantum information, while quantum information is physically transferred from Alice to Bob via photonic states being the excellent long-distance carriers of quantum information.

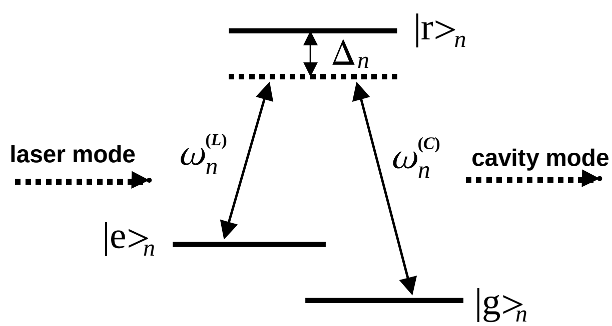

FIG. 4: Energy-level configuration of the atom trapped in Alice's cavity ($n = a$) or Bob's cavity ($n = b$). Key: $|g\rangle_n$, $|e\rangle_n$ – atomic levels for the information storage; $|r\rangle_n$ – atomic excited level; $\omega_n^{(C)}$ – frequency of the cavity field; $\omega_n^{(L)}$ – frequency of the classical laser field; Δ_n – detuning.

the evolution of the lower levels. Thus, we can assume that the quantum information is stored only in two levels $|g\rangle_n$ and $|e\rangle_n$. Both Alice's and Bob's cavities initially have no photons being described by vacuum state $|0\rangle_n$, and Bob's atom is initially in state $|e\rangle$. Alice does not know her atomic state, which is of the form $|\psi\rangle_a = c|g\rangle_a + c'|e\rangle_a$ (with the unknown coefficients c and c' such that $|c|^2 + |c'|^2 = 1$). The main task is to teleport the state $|\psi\rangle_a$ to Bob. First, as a preparation of the state, Alice maps the atomic state $|\psi\rangle_a$ on her cavity mode by illuminating the atom A_a with the laser L_a for a proper period of time. In the meantime, Bob illuminates his atom A_b with the laser L_b for another appropriate time period to generate an atom–cavity-field entangled state $|\Psi\rangle_b = 2^{-1/2}(|e\rangle_b|0\rangle_b + i|g\rangle_b|1\rangle_b)$, where $|1\rangle_b$ and $|0\rangle_b$ stand for the cavity mode state with one or no photons, respectively. Alice and Bob should synchronize their actions to finish simultaneously the preparations of their states since photons are leaking out from both the cavities. Those photons are mixed on the 50-50 beam splitter BS. Cavities are assumed to be single-sided so that the only leakage of photons occur through the sides of the cavities facing BS. The next step is the detection of the photons, when Alice just waits for a finite time period for click of the photon counter either D_1 or D_2. This joint detection of photons leaking from distinct cavities C_a or C_b constitutes a measurement that enables a disembodied transfer of quantum information from Alice's atom A_a to Bob's atom A_b. The cases, when Alice registers no clicks or two clicks, are rejected as the failure of the teleportation. At the post detection stage, Bob applies to the transferred state a proper phase shift depending on whether detector D_1 or D_2 clicked. This step corresponds to the unitary transformation U described in figure 1 and completes the teleportation protocol.

It is worth noting that the presented scheme, like the quantum scissors, is probabilistic in the sense that the original state is destroyed even if the teleportation fails, which is the case when photon counters do not register one photon. However, the scheme can be modified to a teleportation protocol with insurance by entangling the initial Alice's atom A_a with a reserve atom A_r also trapped in her cavity C_a [11].

IV. ENTANGLEMENT SWAPPING

Quantum teleportation of qubit states is one of the most fundamental protocols of quantum communication. The generalized version of the standard QT protocol is the *entanglement swapping* [1, 12], where an entangled state of qubit is teleported, as explained in figure 5. The QT and entanglement swapping are essential parts of any quantum communication toolbox. To show the similarities between the protocols we have used the same symbols in figures 1 and 5. There are two equivalent ways to interpret teleportation in the scheme: The state of qubit 1 is teleported to qubit 3 or that of qubit 2 is teleported to qubit 4 after additional unitary transformation of the output states of qubits 3 and 4, respectively. The first experiment demonstrating the entanglement swapping was performed also by Zeilinger's group [13]. They succeeded to entangle freely propagating particles that never physically interacted with one another or which have never been dynamically coupled by any other means. This was probably the first direct experimental demonstration that quantum entanglement requires the entangled particles neither to come from a common source nor to have interacted in the past.

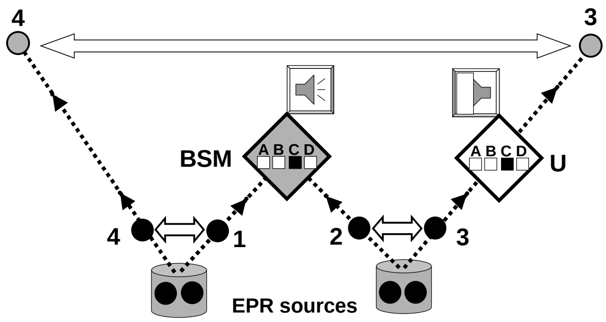

FIG. 5: Principles of the entanglement swapping: Application of teleportation to entangle qubits that never interacted. Two EPR sources produce two pairs of entangled qubits: 1–4 and 2–3 (marked by short arrows). Bell-state measurement (BSM), as explained in figure 1, is performed on two qubits 1 and 2, one from each pair. This measurement entangles outgoing qubits 3 and 4 (marked by long arrow).

V. ENTANGLEMENT DISTILLATION

As emphasized in all our discussions, EPR pair is an essential resource for teleportation. So far, we have assumed that Alice and Bob share the EPR states that can be accurately represented by for instance $|\Phi_A\rangle$ in Eq. (1). Since Alice and Bob cannot create the EPR pairs by classical communication, in order to share the EPR pairs, distribution processes are required. Suppose that Alice creates the EPR pairs and sends half of the pairs to Bob. In realistic situations, due to the noise in transmission channel or the imperfections of Alice's and Bob's devises, the shared pairs are not the same as the ones represented by $|\Phi_A\rangle$. Thus, for the QT protocol or other applications, conversions from these imperfect pairs (less entangled pairs) to the EPR pairs are needed. This process is called the *entanglement distillation* and so far many theoretical proposals on physical realization of the process have been suggested [14]. These proposals allow Alice and Bob to distill the EPR pairs from less entangled pairs by means of the operations performed separately by Alice and Bob, and the classical communication. In 2001, the joint groups of Paul Kwiat of Los Alamos National Laboratory and of Nicolas Gisin of the University of Geneva [15] have first demonstrated experimental distillation of the EPR pairs by local filtering, which is an operation individually applied to each distributed pair. Recently, Nobuyuki Imoto's group at SOKENDAI [16] successfully extracted an entangled photon pair from two identically less entangled pairs by collective operations, which are the operations applied to two distributed pairs. This is the first experiment that involves the collective operations. These experiments are a step towards experimental realizations of more complicated applications in quantum information theory.

VI. PERSPECTIVES AND CONCLUSIONS

In Zeilinger's opinion, quantum teleportation between atoms separated at macroscopic distances can experimentally be realized within a few years and between molecules within a decade or so. Nielsen's et al. [9] experimental teleportation of nuclear-spin states although over microscopic distances is a good prognostic. But, probably, the most spectacular and promising is the experiment of Eugene Polzik's group of the University of Aarhus [5] realizing the quantum entanglement between macroscopic objects, i.e., a pair of caesium gas clouds containing 10^{12} atoms each. Even though, the two samples were just millimeters apart, they could in principle be entangled at much longer distances. Entanglement of such large objects enables 'bulk' properties, like collective spin, to be teleported from one gas cloud to another. Thus, the Polzik experiment, possibly, opens the way for quantum teleportation between macroscopic atomic objects.

The natural question arises what is the greatest difficulty in teleporting people or other macroscopic objects. According to quantum teleportation protocol, information from every tiny particle in a person should be extracted, transferred to particles elsewhere and assembled into an exact replica of the person. And the problem is that human body is composed of about 10^{27} of atoms and sending information about each individual atom state (not only about collective properties as was done in Polzik's experiment) would require, by applying the up-to-date technology, time longer than that of the Universe. Thus, as emphasized by scientists, anything even approximating quantum teleportation of complex living beings, even bacteria or virus, is completely beyond our technological capabilities.

Despite this unrealistic dream of new means of transportation, quantum teleportation is not only a trick but plays one of the key roles in quantum information research of the last decade. Without any doubt, it has already become an essential tool in quantum computers [17], which in turn enable, if constructed, superfast calculations for simulation of the Universe, weather forecasts or artificial intelligence. But physicists must humbly admit that they only begin to understand why, in fact, the quantum teleportation is possible in our quantum world.

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