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Kerr nonlinear coupler and entanglement

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Abstract
We discuss a model comprising two coupled nonlinear oscillators (Kerr-like nonlinear coupler) with one of them pumped by an external coherent excitation. Applying the method of nonlinear quantum scissors we show that the quantum evolution of the coupler can be closed within a finite set of n-photon Fock states. Moreover, we show that the system is able to generate Bell-like states and, as a consequence, the coupler discussed behaves as a two-qubit system. We also analyse the effects of dissipation on entanglement of formation parametrized by concurrence.

Keywords: entanglement, Kerr coupler, Kerr nonlinearity, concurrence, Bell states

1. Introduction

Quantum entanglement seems to be one of the most striking phenomena of quantum physics. It is not only one of the most fundamental concepts of quantum information theory, but also allows investigation of many features of nonlocal properties of quantum systems [1]. Various aspects of the entanglement and its generation have been discussed in numerous papers, especially from the point of view of quantum information applications including quantum key distribution [2], superdense coding [3], quantum teleportation [4], fast quantum computations [5, 6], entanglement-assisted communication [7] or broadcasting of entanglement [8].

In this paper we shall concentrate on the dynamics of the *Kerr nonlinear coupler* and its ability to produce quantum entangled states. Since the pioneering works of Jensen [9] and Maier [10], the nonlinear couplers have attracted an increasing interest [11–17] (for reviews see [18, 19]). As shown in classical [9, 10] and quantum [11] models, the Kerr couplers can exhibit self-trapping, self-modulation and self-switching of the energy of the coupled modes. These phenomena have potential applications in optical communications as, e.g., intensity-dependent routing switches. Among various other quantum statistical properties, it has been shown that the Kerr couplers can be a source of sub-Poissonian and squeezed light [12–16]. Another group of papers concerns the correspondences between the quantum and classical dynamics of such systems [11] and their chaotic dynamics, including synchronization effects [17].

Quantum optical systems based on Kerr nonlinearity have been applied for various quantum information purposes including entanglement purification [20], complete quantum teleportation [21], or realization of qubit phase gates [22]. Here, we present another simple quantum information application of Kerr nonlinearities, namely for generation of entangled optical qubits from classical light.

We are interested here in a simple model comprising two quantum nonlinear oscillators located inside one cavity. These oscillators are linearly coupled to each other, while one of the oscillators is excited by an external coherent field of a constant amplitude. For this model we shall answer the questions of whether it is possible to close the dynamics of the excited nonlinear coupler within a finite set of n-photon states and, which is the main subject of this paper, whether a nonlinear excited coupler can be a source of maximally entangled (ME) states.

2. The model and solutions

The model of the Kerr nonlinear coupler discussed here contains two nonlinear oscillators linearly coupled to each other and, additionally, one of them is coupled to an external coherent field as presented in figure 1. We assume that this excitation is linear and has a constant amplitude. This system

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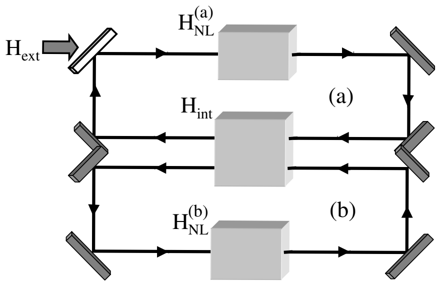

Figure 1. Scheme of a pumped nonlinear coupler, described by Hamiltonian (1), implemented by two ring cavities (a and b) filled with Kerr media, where cavity a is being excited by a single-mode external classical field.

can be described by the following Hamiltonian:

$$\hat{H} = \hat{H}_{\rm NL} + \hat{H}_{\rm int} + \hat{H}_{\rm ext} \tag{1}$$

where

$$\hat{H}_{\rm NL} \equiv \hat{H}_{\rm NL}^{(a)} + \hat{H}_{\rm NL}^{(b)} = \frac{\chi_a}{2}(\hat{a}^\dagger)^2\hat{a}^2 + \frac{\chi_b}{2}(\hat{b}^\dagger)^2\hat{b}^2, \tag{2}$$

$$\hat{H}_{\rm int} = \epsilon\hat{a}^\dagger\hat{b} + \epsilon^*\hat{a}\hat{b}^\dagger, \tag{3}$$

$$\hat{H}_{\rm ext} = \alpha\hat{a}^\dagger + \alpha^*\hat{a}. \tag{4}$$

We see that $\hat{H}_{\rm NL}$ describes nonlinear oscillators, $\hat{H}_{\rm int}$ corresponds to an internal coupling, whereas the term $\hat{H}_{\rm ext}$ describes a linear coupling between the external field and the mode of the field inside our cavity corresponding to the oscillator a. The parameters χ_a and χ_b are nonlinearity constants of the oscillators a and b, respectively, ϵ describes the strength of the oscillator–oscillator coupling, whereas α is the strength of the external excitation of the oscillator a. It is worth noting that our Hamiltonian $\hat{H}_{\rm int}$ does not include nonlinear coupling proportional to $\hat{b}^\dagger\hat{b}\hat{a}^\dagger\hat{a}$ but only the linear one, described by (3). Nevertheless, the same Hamiltonian $\hat{H}_{\rm NL}+\hat{H}_{\rm int}$ as ours was used, e.g., by Bernstein [23] and Chefles and Barnett [11] to describe the nonlinear coupler.

In the first part of our analysis we neglect damping processes in our model, thus the system evolution can be described by a time-dependent wavefunction. This function can be written in the n-photon Fock basis as

$$|\psi(t)\rangle = \sum_{n,m=0}^{\infty} c_{n,m}(t)|n\rangle_a|m\rangle_b \tag{5}$$

where $c_{n,m}(t)$ is a complex probability amplitude of finding our system in the n-photon and m-photon states for mode a and b, respectively.

We have included here an external coupling and, therefore, the energy inside the cavity is not conserved. As a consequence, we can expect that in the evolution of the system many of the states corresponding to a high number of photons will be involved. However, we can overcome this difficulty by applying the *nonlinear quantum scissors* method discussed in [24] (for discussion concerning quantum states defined in finite-dimensional Hilbert spaces and the methods of their generation see the review papers [25, 26] and references therein). Namely, it is seen from the form of $\hat{H}_{\rm NL}$ that this Hamiltonian produces degenerate levels of the energy equal to zero, corresponding to the following four states: $|0\rangle_a|0\rangle_b$, $|1\rangle_a|0\rangle_b$, $|0\rangle_a|1\rangle_b$ and $|1\rangle_a|1\rangle_b$. Moreover, all couplings discussed here have constant envelopes, and, similarly as in [24], we assume that they are weak. Therefore, we can treat transitions within the mentioned set of the states as of resonant nature. The evolution of the discussed system is closed within the set of these four states and interactions with other states can be neglected in our approximation. Thus, the wavefunction describing our model can be written in the form

$$\begin{aligned}|\psi(t)\rangle &= c_{0,0}(t)|0\rangle_a|0\rangle_b + c_{1,0}(t)|1\rangle_a|0\rangle_b \\ &\quad + c_{0,1}(t)|0\rangle_a|1\rangle_b + c_{1,1}(t)|1\rangle_a|1\rangle_b\end{aligned} \tag{6}$$

and, hence, the equations of motion for the system are

$$\begin{aligned}{\rm i}\frac{\rm d}{{\rm d}t}c_{0,0} &= \alpha^* c_{1,0},\\ {\rm i}\frac{\rm d}{{\rm d}t}c_{1,0} &= \epsilon c_{0,1} + \alpha c_{0,0},\\ {\rm i}\frac{\rm d}{{\rm d}t}c_{0,1} &= \epsilon^* c_{1,0} + \alpha^* c_{1,1},\\ {\rm i}\frac{\rm d}{{\rm d}t}c_{1,1} &= \alpha c_{0,1}.\end{aligned} \tag{7}$$

To solve these equations we need to find roots of the fourth-order polynomial, which leads to a very complicated and unreadable form of final formulae. Therefore, although it is formally possible, we shall not write the analytical solutions in their most general form and we shall restrict our considerations to the case of real $\alpha = \epsilon$. Moreover, we assume that for the time $t = 0$ both oscillators are in vacuum states, i.e.,

$$|\psi(t=0)\rangle = |0\rangle_a|0\rangle_b. \tag{8}$$

Then we get the following solutions for the probability amplitudes $c_{i,j}$ $(i, j = 0, 1)$:

$$\begin{aligned}c_{0,0}(t) &= \cos(xt)\cos(yt) + \frac{1}{\sqrt{5}}\sin(xt)\sin(yt),\\ c_{1,0}(t) &= -{\rm i}\frac{2}{\sqrt{5}}\cos(xt)\sin(yt),\\ c_{0,1}(t) &= -\frac{2}{\sqrt{5}}\sin(xt)\sin(yt),\\ c_{1,1}(t) &= {\rm i}\left[\frac{1}{\sqrt{5}}\cos(xt)\sin(yt) - \sin(xt)\cos(yt)\right]\end{aligned} \tag{9}$$

where $x = \alpha/2$ and $y = \sqrt{5}x$. Solution (9) is valid under the condition $\chi_j \gg \epsilon = \alpha$ $(j = a, b)$, which implies that it is apparently independent of nonlinearities χ_j. But it should be stressed that the corresponding nonlinear Hamiltonian $\hat{H}_{\rm NL}$, given by (2), is responsible for the truncation of the infinite-dimensional state to the finite superposition, given by (6). Otherwise, if χ_j were not much stronger than ϵ and α, the state generated would not be truncated to the finite superposition (6) and the probability amplitudes $c_{n,m}(t)$ would depend explicitly on nonlinearities χ_j.

To check our solutions we can calculate the probability amplitudes numerically in a basis expanded to the states corresponding to greater number of photons than discussed here (for the model discussed our considerations are restricted

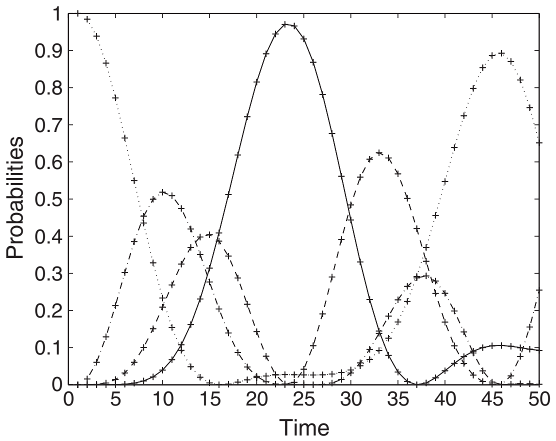

Figure 2. Probabilities for finding the coupler in the $|0\rangle_a|0\rangle_b$ (dotted curve), $|1\rangle_a|0\rangle_b$ (dashed–dotted curve), $|0\rangle_a|1\rangle_b$ (dashed curve) and $|1\rangle_a|1\rangle_b$ (solid curve) states from the analytical results and their numerical counterparts (cross marks). The nonlinearity coefficients $\chi_a = \chi_b = 25$ and the coupling strengths $\epsilon = \alpha = \pi/25$.

by the resonances to the one-photon and vacuum states only). We perform the calculations following the method discussed in [26], and first we construct the unitary evolution operator $\hat{U}$ applying the full Hamiltonian shown in (1):

$$\hat{U} = \exp(-\mathrm{i}\hat{H}t). \tag{10}$$

Then we are able to obtain the wavefunction $|\psi(t)\rangle$ by acting the operator $\hat{U}$ on the initial state of the system, and for the case discussed here we have

$$|\psi(t)\rangle = \hat{U}(t)|0\rangle_a|0\rangle_b. \tag{11}$$

Figure 2 shows both analytical and numerical results of our calculations. We see very good agreement between these two methods, so the model based on the resonances works very well. Moreover, from the numerical results for the probabilities corresponding to the states $|0\rangle_a|2\rangle_b$ (figure 3(a)) and $|1\rangle_a|2\rangle_b$ (figure 3(b)), we see that the states corresponding to the numbers of photons higher than one are practically unpopulated. It is worth mentioning that our numerical calculations have been performed in the m-dimensional Fock basis, where $m \simeq 20$ for each subspace associated with a single mode of the field.

3. Coupler and entanglement

The time evolution of the probability amplitudes can give some information concerning entanglement in our system too. For instance, if we see in a figure that the probability corresponding to one of the discussed states is equal to another one and, additionally, both are equal to 1/2 ($|c_{i,j}|^2 = |c_{k,l}|^2 = 0.5$ for every i, j, k, l), we know that the system generates ME states. Obviously, this method of finding entangled states is not very accurate, especially for the case when we should observe and compare various and often rapidly oscillating probabilities. Therefore, we apply another method convenient for finding entanglement in the system. Namely, we shall express the obtained wavefunction in the Bell basis,

$$|\psi\rangle = b_1|B_1\rangle + b_2|B_2\rangle + b_3|B_3\rangle + b_4|B_4\rangle \tag{12}$$

where the states $|B_i\rangle$, $i = 1, 2, 3, 4$ are Bell-like states that can be expressed as functions of the n-photon states discussed here (Bell-like states differ from the commonly discussed Bell states in the existence of the phase factor—for the case discussed here, one of the n-photon states is multiplied by i):

$$\begin{aligned}
|B_1\rangle &= \frac{1}{\sqrt{2}}(|1\rangle_a|1\rangle_b + \mathrm{i}|0\rangle_a|0\rangle_b),\\
|B_2\rangle &= \frac{1}{\sqrt{2}}(|0\rangle_a|0\rangle_b + \mathrm{i}|1\rangle_a|1\rangle_b),\\
|B_3\rangle &= \frac{1}{\sqrt{2}}(|0\rangle_a|1\rangle_b - \mathrm{i}|1\rangle_a|0\rangle_b),\\
|B_4\rangle &= \frac{1}{\sqrt{2}}(|1\rangle_a|0\rangle_b - \mathrm{i}|0\rangle_a|1\rangle_b).
\end{aligned} \tag{13}$$

These states are ME states, and therefore, for the cases when $|b_i|^2 = 1$, $i = 1, 2, 3, 4$, our system also evolves into an ME state. Figure 4 shows probabilities corresponding to the Bell-like states as a function of time. Moreover, all parameters describing our system are identical to those of figure 2. We see that for the time $t \simeq 115$ we get the state $|B_1\rangle$ and for $t \simeq 80$ the state $|B_2\rangle$ is generated with high accuracy—our system becomes maximally entangled. This entanglement involves the states $|0\rangle_a|0_b\rangle$ and $|1\rangle_a|1_b\rangle$. Of course, one should keep in mind that plots in figure 4 are for the probabilities, not for their complex amplitudes, and hence we get the Bell-like states from (13) with some phase factor. Nevertheless, our states are maximally entangled. Moreover, figure 4 shows that the values of probabilities for the states $|B_3\rangle$ and $|B_4\rangle$ can maximally reach 0.8. As a consequence, the states $|1\rangle_a|0_b\rangle$ and $|0\rangle_a|1_b\rangle$ cannot be maximally entangled for the initial vacuum states $|0\rangle_a|0_b\rangle$. But generation of $|B_3\rangle$ and $|B_4\rangle$ would be possible by assuming that the system is initially in the states $|1\rangle_a|0_b\rangle$ or $|0\rangle_a|1_b\rangle$.

The Bell-like states (13) are maximally entangled; however, they are not the only entangled states that could be produced by the system. Therefore, to measure the entanglement degree of the system we apply the measure that is referred to as the *concurrence*. This quantity proposed by Wootters [27] is one of the most commonly applied measures of the entanglement. The concurrence for two-qubit states is defined as

$$\mathcal{C} = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\} \tag{14}$$

where λ_i ($i = 1, \ldots, 4$) are the square roots of the eigenvalues of the matrix

$$\tilde{\rho} = \rho\left(\sigma_y^a \otimes \sigma_y^b\right)\rho^*\left(\sigma_y^a \otimes \sigma_y^b\right) \tag{15}$$

where $\sigma_y^{\{a,b\}}$ are Pauli matrices defined in subspaces corresponding to the modes $\{a, b\}$, and the eigenvalues λ_i appearing in (14) should be taken in decreasing order. Concurrence takes values from zero to unity, where for unentangled states it vanishes, whereas for ME states it is equal to unity.

Damping is the main and unavoidable source of decoherence which can easily destroy entangled states. Hence, for our results to be applicable in real physical systems, we present a numerical analysis of the damping effects on the concurrence. Let us assume that the leakage of photons from

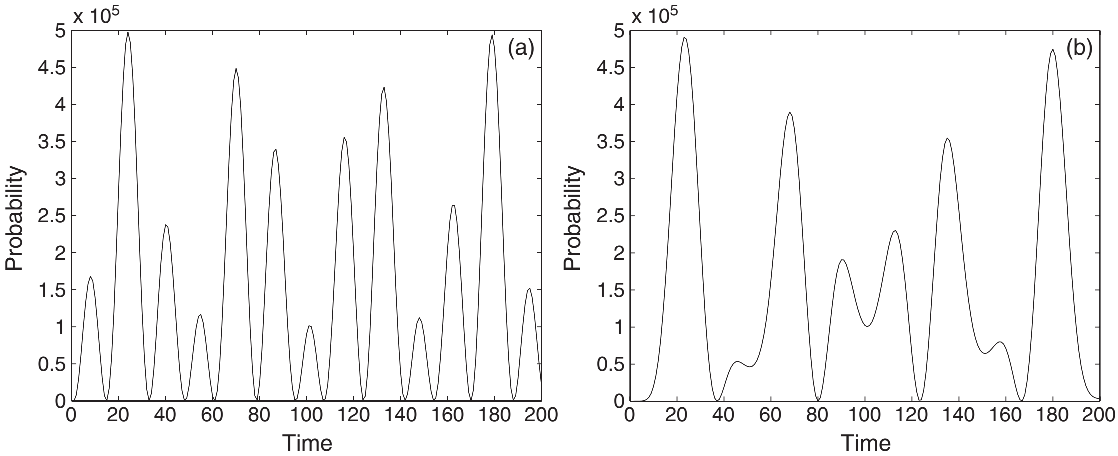

Figure 3. The same as in figure 2 but for the states (a) $|0\rangle_a|2\rangle_b$ and (b) $|1\rangle_a|2\rangle_b$.

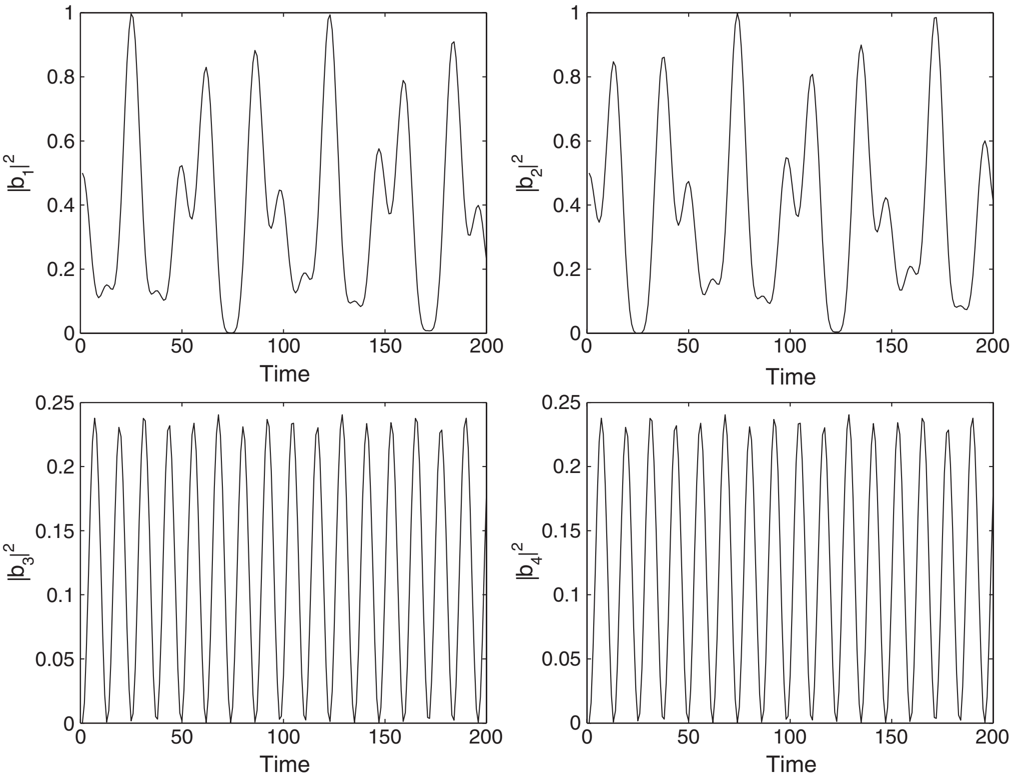

Figure 4. Probabilities for finding the coupler in the Bell-like states. The nonlinearity coefficients χ_a and χ_b, and the coupling strengths ϵ and α, are identical to those of figures 2 and 3.

the cavities a and b is described by the rates κ_a and κ_b, respectively. Starting from Hamiltonian (1) and defining the collapse operators by $\hat{C}_a = \sqrt{2\kappa_a}\hat{a}$ and $\hat{C}_b = \sqrt{2\kappa_b}\hat{b}$, we can write the time-independent Liouvillian in the standard Linblad form

$$\hat{\mathcal{L}}\hat{\rho} = -\mathrm{i}[\hat{H},\hat{\rho}] + \sum_{j=a,b}(\hat{C}_j\hat{\rho}\hat{C}_j^\dagger - \tfrac{1}{2}(\hat{C}_j^\dagger\hat{C}_j\hat{\rho} + \hat{\rho}\hat{C}_j^\dagger\hat{C}_j)). \quad (16)$$

The evolution of the density matrix $\hat{\rho}(t)$ in the dissipative system can be found numerically as a series of complex exponentials $\exp(\sigma_k t)$ given in terms of the eigenvalues σ_k of the Liouvillian $\hat{\mathcal{L}}$, given by (16).

Figure 5 shows the plot of the concurrence evolution for the case discussed here—we have a single external excitation of the coupler and we assume that all couplings existing in the system are weak ($\alpha = \epsilon = \pi/25$) in comparison to Kerr nonlinearities $\chi_a = \chi_b = 25$. Various curves in figure 5 correspond to concurrence evolutions with different dissipation rates. We see that the time-varying concurrence is modulated by an oscillation of low frequency. As a consequence, several maxima appearing here are of various values. Two of them for dissipation-free evolution (depicted by solid curve), which are the closest to unity, correspond to the formation of Bell-like states $|B_1\rangle$ and $|B_2\rangle$ discussed earlier and shown in figure 4. As

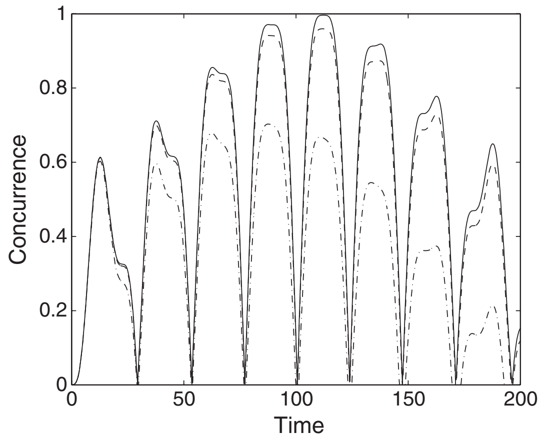

Figure 5. Concurrence for the excited nonlinear coupler for various cavity leakage rates $\kappa_a = \kappa_b$ equal to 0 (solid curve), 10^{-4} (dashed curve) and 10^{-3} (dot–dashed curve). The quantities χ_a, χ_b, ϵ and α are identical to those of figure 4.

a consequence, we can treat our system as a source of ME states for low dissipation. On the scale of figure 5 and for the chosen coupling parameters, the differences between the evolution with the leakage rates $\kappa_a = \kappa_b \leqslant 10^{-5}$ and the dissipation-free evolution are invisible. However, higher leakage rates beyond short time evolution cause essential deterioration of the concurrence, limiting the effective generation of ME states. Thus, the results of our calculations indicate that the system discussed is highly sensitive to the dissipation processes. Even relatively small losses from the cavity are able to destroy the entanglement. Therefore, we should assume that we deal here with a very high Q cavity that is capable of preserving practically the whole radiation field located inside. However, this assumption is very desirable from our point of view. For this case the coupler can be weakly excited by external fields only—the less photons can escape from the cavity through the mirror, the smaller the number of photons that can be injected inside this way.

4. Conclusions

In this paper we have discussed a model of a nonlinear coupler linearly excited by a single-mode coherent field. We have shown that the evolution of the system is closed within a finite set of states and only $|i\rangle_a|j\rangle_b$ $(i, j = 0, 1)$ states are populated. We have applied here the method used for the *nonlinear quantum scissors* [24] and have found some analytical formulae for the probability amplitudes corresponding to these states. We have shown that, starting from the vacuum state $|0\rangle_a|0\rangle_b$ of our system, its evolution leads to Bell-like state generation. Moreover, we have calculated the concurrence and its behaviour indicates that the ME states are produced for the system if the photon leakage rates out of the cavities are less than 10^{-5} for the chosen coupling and nonlinearity parameters. Moreover, we have shown that the concurrence exhibits some modulation effect as a result of the existence of various couplings in our system. For each of the couplings we have a frequency and their interference leads to some long-frequency oscillations in the system.

We see that our model, despite its simplicity, exhibits intriguing features. We can say that the properties of the system discussed here are much desired from the point of view of the physical properties of the nonlinear couplers. Our scheme can be used for generation of entangled optical qubits from classical light, which is a basic but rather simple quantum information problem. Introduction of conditional measurement in this scheme, along the lines of the proposal by Duan *et al* [20], is probably worth further study from the point of view of more sophisticated quantum information applications [20–22].

Finally, we mention the experimental feasibility of the presented scheme. Since our solution is applicable only when the cavity-field intensities are very small, an objection arises that the Kerr nonlinearities are usually negligible in this case. However, the recent breakthrough advances in nonlinear optics involving very weak light fields show that the nonlinearities can be enhanced by several orders of magnitude in ultracold atomic systems using electromagnetically induced transparency when resonant optical absorption is eliminated ([28] and references therein). In particular, giant Kerr nonlinearities have been theoretically predicted [29] and first experimentally measured to be $\sim 10^6$ greater than those in the conventional optical materials [30]. Thus, we believe that the scheme discussed here can be feasible experimentally.

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