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Dissipation in systems of linear and nonlinear quantum scissors

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Abstract

We analyse the truncation of coherent states up to a single-photon Fock state by applying linear quantum scissors, utilizing the projection synthesis in a linear optical system, and nonlinear quantum scissors, implemented by periodically driven cavity with a Kerr medium. Dissipation effects on optical truncation are studied in the Langevin and master equation approaches. Formulae for the fidelity of lossy quantum scissors are found.

Keywords: quantum state engineering, Kerr effect, qubit generation, finite-dimensional coherent states

1. Introduction

The breathtaking advances in quantum computation and quantum information processing in the last decade [1] have stimulated progress in quantum optical state generation and engineering [2]. Among various schemes, the proposal of Pegg, Phillips and Barnett [3, 4] of optical state truncation via projection synthesis has attracted considerable interest [5–20] due to the simplicity of the scheme to generate and teleport 'flying' qubits defined as a running wave superposition of zero- and single-photon states. The scheme is referred to as *linear quantum scissors* (LQS) since the coherent state entering the system is truncated in its Fock expansion to the first two terms using only linear optical elements and performing conditional photon counting. The optical state truncation can also be realized in systems comprising nonlinear elements including a Kerr medium [14, 15]. Such systems will be referred to as *nonlinear quantum scissors* (NQS). Both LQS [12, 13] and NQS [16, 17] can be generalized for the generation of a superposition of N states. It is worth noting that there are fundamental differences between the states truncated by the LQS and NQS [18].

In this paper we analyse the effects of dissipation on state truncation by quantum scissors. Various kinds of losses in quantum scissors have already been analysed, including inefficiency and dark counts of photodetectors [4, 7–10], non-ideal single-photon sources [9, 10], mode mismatch [11], and losses in beam splitters [7]. Özdemir *et al* [9–11] demonstrated that the LQS exhibit surprisingly high fidelity in realistic setups even with conventional photon counters, so long as the amplitude of the input coherent state is sufficiently small. LQS have recently been realized experimentally by Babichev *et al* [5] and Resch *et al* [6], although only in the low-intensity regime. The effect of losses on the optical truncation in NQS has been studied for a zero-temperature reservoir [19] and imperfect photodetection [15].

These studies of losses in LQS (with few exceptions, e.g., for [7]) have been based on the quantum detection and estimation theory using the positive operator valued measures (POVM) [21]. In quantum-optics textbooks (see, e.g., [22, 23]), the quantum-statistical properties of dissipative systems are usually treated in three ways, by applying

(i) the Langevin (Langevin–Heisenberg) equations of motion with stochastic forces,
(ii) the master equation for the density matrix, and
(iii) the classical Fokker–Planck equation for quasiprobability distribution.

In the next section we will apply the Langevin approach to describe dissipative LQS, while in section 3 we shall use the master equation approach to study dissipative NQS.

2. Lossy linear quantum scissors in the Langevin approach

The linear quantum scissors device of Pegg, Phillips and Barnett [3, 4] is a simple physical system for optical state

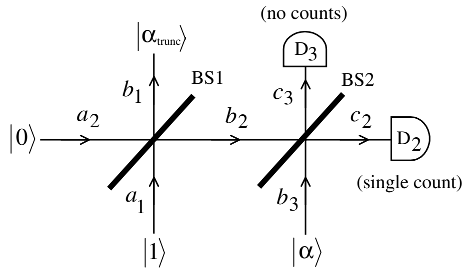

Figure 1. The scheme of linear quantum scissors: $|\alpha\rangle$ is the input coherent state, $|\alpha_{\rm trunc}\rangle$ is the output truncated coherent state, $|0\rangle$ and $|1\rangle$ are vacuum and single-photon states, respectively; BS1 and BS2 are beam splitters; D2 and D3 are photon detectors. For a successful truncation process, one of the detectors should detect one photon, while the other should detect no photons.

truncation based only on *linear* optical elements (two beam splitters BS1 and BS2) and two photodetectors (D2 and D3) as depicted in figure 1. If the input modes a_1 and a_2 are in the single-photon and vacuum states, respectively, and one photon is detected at D2 but no photons at D3, then the lossless LQS device with 50/50 beam splitters truncates the input coherent state $|\alpha\rangle$ in mode b_3 to the following superposition of vacuum and single-photon states in mode b_1:

$$|\alpha_{\rm trunc}\rangle_{b1} = N'{}_{c2c3}\langle 10|\psi_{\rm out}\rangle_{b1c2c3} = \frac{|0\rangle_{b1} + \alpha|1\rangle_{b1}}{\sqrt{1+|\alpha|^2}} \qquad (1)$$

where α is the complex amplitude and N' is a renormalization constant. The state (1) is referred to as the truncated two-dimensional (or two-level) coherent state since it is the normalized superposition of the first two terms of the Fock expansion of the Glauber coherent state. By introducing a new variable $\bar\alpha$ such that $\cos(|\bar\alpha|) = 1/\sqrt{1+|\alpha|^2}$ and $\sin(|\bar\alpha|) = |\alpha|/\sqrt{1+|\alpha|^2}$, and $\varphi = {\rm Arg}\,\alpha$, state (1) can be rewritten as

$$|\alpha_{\rm trunc}\rangle = \cos(|\bar\alpha|)|0\rangle + {\rm e}^{{\rm i}\varphi}\sin(|\bar\alpha|)|1\rangle \qquad (2)$$

where, for brevity, subscript b_1 is omitted. If the jth ($j = 1, 2$) beam splitter has an arbitrary but real transmission coefficient t_j and an imaginary reflection coefficient r_j, then the LQS generates the state [9]

$$|\psi\rangle_{b1} = \frac{|r_1t_2||0\rangle_{b1} + \alpha|r_2t_1||1\rangle_{b1}}{\sqrt{|r_1t_2|^2 + |\alpha|^2|r_2t_1|^2}}. \qquad (3)$$

This state evolves into the truncated coherent state (1) by assuming identical BSs ($r_1 = r_2$ and $t_1 = t_2$).

In general, the transmission and reflection coefficients of a perfect BS obey the conditions $|t|^2 + |r|^2 = 1$ and $tr^* + t^*r = 0$, implied by the unitarity of BS transformation. By including dissipation, these conditions can be violated. Thus, the main goal of this section is to analyse the deterioration of the truncation process due to the noise introduced by lossy beam splitters and also by inefficient photodetectors. In the simplest approach, one can model the BS losses and finite detector efficiency by adding to our system additional beam splitters, then all components of the system (including the new BSs) can be assumed perfect. Here, we apply another standard approach of the quantum theory of damping based on the Langevin noise operators [22, 23]. We follow the analyses of Barnett *et al* [24, 25] and Villas-Bôas *et al* [7]. The lossy BS1 transforms the input annihilation operators $\hat a_j$ into the output $\hat b_j$ as follows [24, 25]:

$$\begin{aligned}\hat a_1 &= t_1^*\hat b_1 + r_1^*\hat b_2 + \hat L_{a1},\\ \hat a_2 &= r_1^*\hat b_1 + t_1^*\hat b_2 + \hat L_{a2}\end{aligned} \qquad (4)$$

where we use the notation of figure 1, and $\hat L_{a1}$ and $\hat L_{a2}$ are the Langevin noise (force) operators satisfying the following commutation relations:

$$\begin{aligned}[\hat L_{a1}, \hat L_{a1}^\dagger] &= [\hat L_{a2}, \hat L_{a2}^\dagger] = 1 - |t_1|^2 - |r_1|^2 \equiv \Gamma_1,\\ [\hat L_{a1}, \hat L_{a2}^\dagger] &= [\hat L_{a2}, \hat L_{a1}^\dagger] = -(t_1r_1^* + t_1^*r_1) \equiv -\Omega_1.\end{aligned} \qquad (5)$$

The transformation between the input ($\hat b_j$) and output ($\hat c_j$) annihilation operators of the lossy BS2 together with the effect of finite efficiency ($\eta \equiv \eta_1 = \eta_2$) of the detectors generalizes to [7]

$$\begin{aligned}\hat b_2 &= \sqrt{\eta}t_2^*\hat c_2 + \sqrt{\eta}r_2^*\hat c_3 + \hat L_{b2},\\ \hat b_3 &= \sqrt{\eta}r_2^*\hat c_2 + \sqrt{\eta}t_2^*\hat c_3 + \hat L_{b3}\end{aligned} \qquad (6)$$

where the Langevin noise operators $\hat L_{b2}$ and $\hat L_{b3}$ obey

$$\begin{aligned}[\hat L_{b2}, \hat L_{b2}^\dagger] &= [\hat L_{b3}, \hat L_{b3}^\dagger] = \eta\Gamma_2 + (1-\eta) \equiv x,\\ [\hat L_{b2}, \hat L_{b3}^\dagger] &= [\hat L_{b3}, \hat L_{b2}^\dagger] = -\eta\Omega_2.\end{aligned} \qquad (7)$$

In (5) and (7), $\Omega_j = t_jr_j^* + t_j^*r_j$ and $\Gamma_j = 1 - |t_j|^2 - |r_j|^2$ are the jth beam splitter phase and amplitude dissipation coefficients, respectively, which vanish for perfect beam splitters. For simplicity, we assume that the BSs are identical ($r_1 = r_2 \equiv r$, $t_1 = t_2 \equiv t$) and that they cause only amplitude damping ($\Gamma \equiv \Gamma_1 = \Gamma_2 \neq 0$) without introducing phase noise ($\Omega_1 = \Omega_2 = 0$). By applying the transformations (4) and (6) for the input state $|\psi_{\rm in}\rangle_{a1a2b3} = |1\rangle_{a1}|0\rangle_{a2}|\alpha\rangle_{b3}$ and performing the conditional measurement (projection synthesis) on modes c_2 and c_3 (as shown in figure 1), one finds that the state of the output mode b_1 of the LQS is entangled with the environment as follows [7]:

$$\begin{aligned}|\psi\rangle_{b1E} &= N''{}_{c2c3}\langle 10|\psi_{\rm out}\rangle_{b1c2c3E}\\ &= N(|0\rangle_{b1}|\mathbf{\Lambda}_0\rangle_E + \alpha|1\rangle_{b1}|\mathbf{\Lambda}_1\rangle_E)\end{aligned} \qquad (8)$$

where we write the environmental states compactly as

$$\begin{aligned}|\mathbf{\Lambda}_0\rangle_E &= \sqrt{\eta}r(t + \alpha r\hat L_{a2}^\dagger + \alpha\hat L_{a1}^\dagger)\exp(\alpha\hat L_{b3}^\dagger)|\mathbf{0}\rangle_E,\\ |\mathbf{\Lambda}_1\rangle_E &= \sqrt{\eta}rt\exp(\alpha\hat L_{b3}^\dagger)|\mathbf{0}\rangle_E\end{aligned} \qquad (9)$$

and the normalization N is given by

$$N = \{\eta|r|^2\,|\alpha|^2{\rm e}^{x|\alpha|^2}[|t|^2(|\alpha|^{-2}+1) + |r|^2x + \Gamma]\}^{-1/2} \qquad (10)$$

and N'' is a renormalization constant. The fidelity of the output state (8) of the lossy LQS to a desired perfectly truncated state, given by (1), can be calculated from

$$F \equiv \|{}_{b1}\langle\alpha_{\rm trunc}|\psi\rangle_{b1E}\|^2 \qquad (11)$$

which leads us to the following relation:

$$\begin{aligned}F &= N^2\eta|r|^2\exp(x|\alpha|^2)\\ &\quad\times\left(|t|^2(|\alpha|^2+1) + \frac{|\alpha|^2}{1+|\alpha|^2}(|r|^2x + \Gamma)\right)\end{aligned} \qquad (12)$$

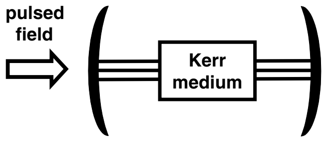

Figure 2. The scheme of nonlinear quantum scissors: a cavity with a Kerr medium is pumped by external ultra-short pulses of laser light.

where the normalization N is given by (10). By defining $R = 1/|\alpha|^2$, equation (12) can be simplified to

$$F = 1 - \frac{(\eta\Gamma + 1 - \eta)|r|^2 + \Gamma}{(1+R)[(\eta\Gamma + 1 - \eta)|r|^2 + \Gamma + |t|^2(1+R)]} \quad (13)$$

where $x = \eta\Gamma + (1 - \eta)$. In a special case for 50/50 BSs, $|r|^2 = |t|^2$, our solution simplifies to that of Villas-Bôas *et al* ([26]; note that the corresponding fidelity in [7] is misprinted). By neglecting the losses caused by the beam splitters, solution (13) is further reduced to the well-known Pegg–Phillips–Barnett fidelity [3]:

$$F = 1 - \frac{|\alpha|^4(1-\eta)}{(1+|\alpha|^2)[1+|\alpha|^2(2-\eta)]}. \quad (14)$$

By also assuming perfect detectors, the fidelity becomes unity, as expected.

3. Lossy nonlinear quantum scissors in the master equation approach

In the nonlinear quantum scissors scheme, schematically depicted in figure 2, a cavity mode is pumped by an external classical pulsed laser field, described by the Hamiltonian $\hat{H}_K$, and is interacting with a Kerr medium, described by the Hamiltonian $\hat{H}_{NL}$. Thus, the whole system Hamiltonian is given by [14, 19]

$$\hat{H}_S = \hat{H}_0 + \hat{H}_{NL} + \hat{H}_K \quad (15)$$

where

$$\hat{H}_{NL} = \hbar\frac{\kappa}{2}(\hat{a}^\dagger)^2\hat{a}^2, \quad (16)$$

$$\hat{H}_K = \hbar\epsilon(\hat{a}^\dagger + \hat{a})\sum_{k=-\infty}^{\infty}\delta(t - kT_K) \quad (17)$$

and the free Hamiltonian of the system is $\hat{H}_0 = \hbar\omega\hat{a}^\dagger\hat{a}$. In equation (16), $\hat{a}$ is the annihilation operator for a cavity mode at frequency ω, and κ is the nonlinear coupling proportional to the third-order susceptibility of the Kerr medium. In (17), Dirac δ-functions describe external ultra-short light pulses (kicks); the real parameter ϵ is the strength of the interaction of the cavity mode with the external field; T_K is the period of free evolution between the kicks. The truncation process in the system, given by (15), occurs if

(i) $T_K \gg T_{\text{round-trip}} \gg 2\pi/\omega$, where ω is the light frequency and $T_{\text{round-trip}}$ is the round-trip time of the light in the cavity, and
(ii) the kicks are much weaker than the Kerr nonlinear interaction, $\epsilon \ll \kappa$.

As shown in [14, 16], the state generated by the NQS is a two-dimensional coherent state [27, 28] of the form

$$|\bar{\alpha}_{\text{trunc}}\rangle \approx \cos(|\bar{\alpha}|)|0\rangle - \text{i}\sin(|\bar{\alpha}|)|1\rangle \quad (18)$$

where $\bar{\alpha} = -\text{i}k\epsilon$. Dissipation of the NQS system is modelled by its coupling to a reservoir of oscillators (heat bath) described by the Hamiltonian

$$\hat{H} = \hat{H}_S + \hat{H}_R + \hat{H}_{SR}, \quad (19)$$

$$\hat{H}_{SR} = \hbar\sum_j(g_j\hat{a}\hat{b}_j^\dagger + g_j^*\hat{a}^\dagger\hat{b}_j) \quad (20)$$

where $\hat{H}_S$ is given by (15) and $\hat{H}_R = \hbar\sum_j \chi_j\hat{b}_j^\dagger\hat{b}_j$ is the free Hamiltonian of the reservoir, where $\hat{b}_j$ is the annihilation operator of the jth reservoir oscillator. By applying the standard methods of the quantum theory of damping [22], one finds that the NQS evolution between the kicks is governed under the Markov approximation by the following master equation in the interaction picture:

$$\frac{\partial}{\partial t}\hat{\rho} = -\text{i}\frac{\kappa}{2}[(\hat{a}^\dagger)^2\hat{a}^2, \hat{\rho}] - \frac{\gamma}{2}([\hat{a}^\dagger, \hat{a}\hat{\rho}] + \text{h.c.}) + \gamma\bar{n}[\hat{a}^\dagger, [\hat{\rho}, \hat{a}]] \quad (21)$$

where γ is the damping constant and $\bar{n}$ is the mean number of thermal photons, $\bar{n} = \{\exp[\hbar\omega/(k_B T)] - 1\}^{-1}$, at the reservoir temperature T, where k_B is the Boltzmann constant. Let the kick be applied at time t_K, then the solution of equation (21) for any time t after t_K but before moment $t_K + T_K$ is the same as the solution for the ordinary damped anharmonic oscillator [29, 31, 32] with the initial state given at time t_K. We can write the solution compactly as ($\rho_{nm} \equiv \langle n|\hat{\rho}|m\rangle$):

$$\rho_{nm}(t_K + t) = \exp\left[\frac{\gamma t}{2} + \text{i}(n-m)\kappa t\right]E_{n-m}^{n+m+1}(t) \times \sum_{l=0}^{\infty}\rho_{n+l,m+l}(t_K)\sqrt{C_n^{n+l}C_m^{m+l}}\,\bar{g}_{n-m}^l(t) \times F\left[-n, -m, l+1; \frac{4\bar{n}(\bar{n}+1)}{\Delta_{n-m}^2}\sinh^2 t_{n-m}\right] \quad (22)$$

where F is the hypergeometric function, C_y^x are binomial coefficients, $t_x = \gamma\Delta_x t/2$, and

$$\bar{g}_x(t) = \frac{2(\bar{n}+1)}{\Omega_x + \Delta_x\coth t_x}, \qquad E_x(t) = \frac{\Delta_x}{\Omega_x\sinh t_x + \Delta_x\cosh t_x} \quad (23)$$

with $\Delta_x = \sqrt{\Omega_x^2 - 4\bar{n}(\bar{n}+1)}$ and $\Omega_x = 1 + 2\bar{n} + \text{i}\kappa x/\gamma$. By assuming the reservoir to be at zero temperature, the solution (22) reduces to [30, 33, 34]

$$\rho_{nm}(\tau_K + \tau) = \exp\left[\text{i}(n-m)\frac{\tau}{2}\right]f_{n-m}^{(n+m)/2}(\tau) \times \sum_{l=0}^{\infty}\rho_{n+l,m+l}(\tau_K)\sqrt{C_n^{n+l}C_m^{m+l}}\left(\frac{\lambda[1 - f_{n-m}(\tau)]}{\lambda + \text{i}(n-m)}\right)^l \quad (24)$$

where τ is the scaled time given by $\tau = \kappa t$, so $\tau_K = \kappa t_K$. Moreover, $\lambda = \gamma/\kappa$, and $f_x(\tau) = \exp[-(\lambda + \text{i}x)\tau]$. For a lossless anharmonic oscillator, i.e. for $\lambda = 0$, the solution (24) further simplifies to

$$\rho_{nm}(\tau_K + \tau) = \exp\left\{\text{i}[n(n-1) - m(m-1)]\frac{\tau}{2}\right\}\rho_{nm}(\tau_K). \quad (25)$$

Solution (22) describes the evolution of the NQS only between the kicks. On the other hand, the evolution at each kick is given by

$$\lim_{\delta\to 0}\langle n|\hat{\rho}(t_{\rm K}+\delta)|m\rangle = \lim_{\delta\to 0}\sum_{n',m'=0}^{\infty} U_{nn'}\langle n'|\hat{\rho}(t_{\rm K}-\delta)|m'\rangle U^{*}_{mm'} \quad (26)$$

where

$$U_{nm} = \langle n|\hat{U}|m\rangle = \langle n|\exp[-{\rm i}\epsilon(\hat{a}^{\dagger}+\hat{a})]|m\rangle \quad (27)$$

in analogy to the Milburn–Holmes transformation for the pulsed parametric amplifier with a Kerr nonlinearity [34]. By observing that $\hat{U}$ is the displacement operator $\hat{U} = \exp[-{\rm i}\epsilon(\hat{a}^{\dagger}+\hat{a})] = \hat{D}(-{\rm i}\epsilon)$, we can use the well-known Cahill–Glauber [35] formulae leading for $n \geqslant m$ to

$$U_{nm} = {\rm e}^{-\epsilon^2/2}\sqrt{\frac{m!}{n!}}(-{\rm i}\epsilon)^{n-m}L_m^{n-m}(\epsilon^2) \quad (28)$$

and for $n < m$ to

$$U_{nm} = {\rm e}^{-\epsilon^2/2}\sqrt{\frac{n!}{m!}}({\rm i}\epsilon)^{m-n}L_n^{m-n}(\epsilon^2) \quad (29)$$

where $L_x^y(z)$ is an associated Laguerre polynomial. Thus, we have a complete solution to describe the effects of dissipation on, in particular, the truncation fidelity after the kth kick, which is given by

$$\bar{F}(t) = \langle\bar{\alpha}_{\rm trunc}|\hat{\rho}(t)|\bar{\alpha}_{\rm trunc}\rangle = \cos^2(k\epsilon)\rho_{00}(t) + \sin(2k\epsilon){\rm Im}\rho_{01}(t) + \sin^2(k\epsilon)\rho_{11}(t) \quad (30)$$

where the perfectly truncated state $|\bar{\alpha}\rangle_{\rm NQS}$ was applied according to (18).

4. Conclusions

We have studied dissipative quantum scissors systems for the truncation of a Glauber (infinite-dimensional) coherent state to a superposition of vacuum and single-photon Fock states (two-dimensional coherent state). We have contrasted the Pegg–Phillips–Barnett quantum scissors based on linear optical elements and the Leoński–Tanaś quantum scissors comprising a nonlinear Kerr medium. We have analysed the effects of dissipation on truncation fidelity in the linear scissors within the Langevin noise operator approach and in the nonlinear system in the master equation approach.

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