Selective truncations of coherent state using projection synthesis

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Abstract: Selective truncation of Fock-state expansion of an optical field can be achieved using projection synthesis. The process removes predetermined Fock states from the input field by conditional measurement and teleportation. We present a scheme based on multiport interferometry to perform projection synthesis. This scheme can be used both as a generalized quantum scissors device (QSD) which filters out Fock states with photon numbers higher than a predetermined value, and also as a quantum punching device (QPD) which selectively removes specific Fock states making holes in the Fock-state expansion of the input field.

Keywords: linear optics, state engineering, projection synthesis, quantum scissors device

INTRODUCTION

Recent theoretical and experimental works have caused an increasing interest in quantum state engineering using linear optics. It has been shown that linear optics can be used for efficient quantum computation, entanglement manipulation and generation of nonclassical optical states [1–6]. Linear optics schemes require single photon generation and detection, beam splitters (BSs) and phase shifters (PS). Parametric down conversion process is exploited to built triggered single photon source, and avalanche photodiodes are used as photon counters to discriminate between the absence and presence of photons. Therefore, such schemes are experimentally realizable with the present level of optics technology.

In this paper, we study a linear optics scheme for quantum state engineering using projection synthesis [7]. Our main interest is employing the scheme to perform the following transformation

$$|\psi\rangle = \sum_{n=0}^{\infty} \gamma_n |n\rangle \longrightarrow |\phi^{(d)}\rangle = \mathcal{N} \sum_{n=0}^{d-1} \gamma_n |n\rangle \qquad (1)$$

where the unknown input optical state $|\psi\rangle$ is truncated to obtain the state $|\phi^{(d)}\rangle$ which is a finite superposition of d states. In Eq. (1), $\mathcal{N}$ is the normalization constant, and will be dropped from the equations in the following, and we will use $\sim$ instead of equality to denote that the state should be normalized. This transformation is achieved by conditional measurement and teleportation process. This process was originally proposed by Pegg, Phillips and Barnett to obtain a superposition state of $d = 2$ of the form $|\phi^{(2)}\rangle \sim \gamma_0|0\rangle + \gamma_1|1\rangle$ by truncating a coherent state $|\psi\rangle = |\alpha\rangle$, and was named as the Quantum Scissors Device (QSD) [7, 8]. Later we have elaborated theoretical treatment of the QSD by proposing an experimentally realizable scheme and discussing how arbitrary superposition states of $d = 2$ can be generated by this simple scheme [9, 10]. The first experiment was performed by Babichev et al.[11]. An extension to $d = 3$ of the original QSD scheme was proposed Koniorczyk et. al by a simple modification of the original QSD scheme [12]. The original QSD scheme is an interesting one because it finds its direct application as a basic element of single-rail version of the linear optical quantum computer. Moreover, it is not only a truncation scheme but also a transportation scheme for superposition states of arbitrary d.

The drawback of the original QSD scheme is that it is enables generation of truncated states up to $d = 3$. In this paper, we propose to use a modified version of the multiport interferometer of Zeilinger et. al [13] which has been experimentally demonstrated [14, 15]. The important difference between the original multiport interferometer and the modified version discuss here is the elimination of the apex BS so that the direct path from the input field to the output field is eliminated. This is crucial for the truncation scheme because we want the process to be done via teleportation.

In the following, we will introduce the generalized QSD scheme based on multiport interferometer and give some examples of the possible truncated states. Then we will discuss how the same scheme can be used as a quantum punching device which eliminates selectively some Fock states from the original superposition state and opening holes in the Fock state expansion by proper choices of conditional measurement and input states.

MULTIPORT INTEREFEROMETER AS QUANTUM SCISSORS DEVICE

A schematic diagram of the eight-port interferometer and the generalized QSD is given in Fig. 1, and the original QSD scheme which can be considered as a six-port interferometer is given in

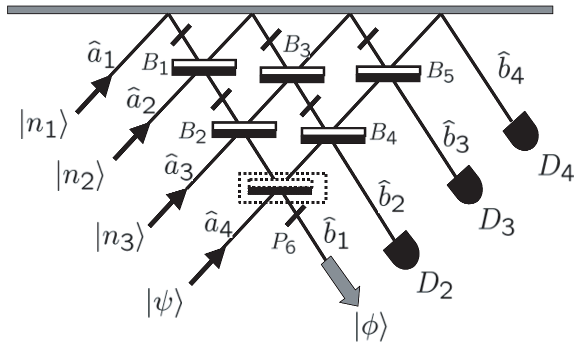

FIG. 1: Generalized eight-port quantum scissors device (QSD). Key: $|\psi\rangle$- input state to be truncated, usually a coherent state $|\alpha\rangle$; $|n_j\rangle$- input Fock states; $|\phi\rangle$- output state, selectively truncated or punched; D_j- photon counters; B_j- beam splitters; P_j- phase shifters; $\hat{a}_j$ and $\hat{b}_j$- input and output annihilation operators, respectively. The beams reflected from the white surface of the beam splitters are π phase shifted while the reflections from the black surface are without any phase shift. The beam splitter drawn with dotted lines and bounded by the dotted box is the beam splitter to be removed to obtain the QSD from the conventional multiport interferometer.

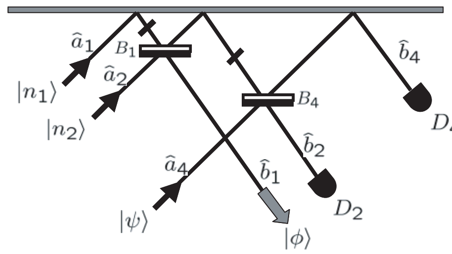

FIG. 2: Representation of the original QSD scheme of Pegg-Phillips-Barnett as a six-port interferometer scheme. Notation is the same as in Fig. 1

Fig. 2. The beamsplitter shown with the dotted lines corresponds to the apex BS that is to be removed from the interferometer to obtain the generalized QSD scheme. If we define the N-mode input state as $|\Psi\rangle$ then the N-mode output state will be $|\Phi\rangle = \hat{U}|\Psi\rangle$ where $\hat{U}$ is the unitary operator describing the evolution of the input state in the interferometer. Denoting the annihilation operators at the input and output ports as column vectors $\hat{\mathbf{a}} \equiv [\hat{a}_1; \hat{a}_2; \cdots; \hat{a}_N]$ and $\hat{\mathbf{b}} \equiv [\hat{b}_1; \hat{b}_2; \cdots; \hat{b}_N]$, respectively, we obtain $\hat{\mathbf{b}} = \hat{U}^\dagger \hat{\mathbf{a}} \hat{U} = \mathbf{S}\hat{\mathbf{a}}$ where $\mathbf{S} = P_6 B_5 P_5 B_4 P_4 B_3 P_3 B_2 P_2 B_1 P_1$ is the scattering matrix obtained by multiplying the scattering matrices of the beamsplitters, B_i and phase shifters P_i used in the scheme from the input to the output. Now considering that at the input port we have the Fock state $|\Psi\rangle = |n_1, \cdots, n_N\rangle \equiv |\mathbf{n}\rangle$, the output state is found as

$$|\Phi\rangle = \hat{U}|\mathbf{n}\rangle = \frac{1}{\sqrt{n_1! \cdots n_N!}} \sum_{\mathbf{j}=1}^{N} \prod_{l=1}^{\nu} S_{j_l x_l} \hat{a}^\dagger_{j_l} |\mathbf{0}\rangle \quad (2)$$

where $S_{j_l x_l}$ are the elements of the unitary scattering matrix $\mathbf{S}$, $\nu = \sum_i n_i$ is the total number of photons, and $\sum_{\mathbf{j}}$ stands for the multiple sum over $j_1, j_2, \cdots, j_\nu$.

Selective State Truncation: In a truncation scheme, what we are interested is to obtain a superposition state by truncating an input coherent state. Therefore, in the generalized QSD scheme (multiport interferometer) we consider the state $|\psi\rangle$ as one of the inputs. In that case, for the eight-port interferometer we can write the input state as $|\Psi\rangle = |n_1\rangle_1 |n_2\rangle_2 |n_3\rangle_3 |\psi\rangle_4$. Now assume that the detectors at the output ports detects N_2, N_3 and N_4 photons whose sum is the total number of photons input into the interferometer, and satisfies the relation $N_2 + N_3 + N_4 = n_1 + n_2 + n_3 = d - 1$. This means that we project the total output state $|\Phi\rangle_{1,2,3,4}$ onto the detected states $|N_2\rangle_2 |N_3\rangle_3 |N_4\rangle_4$. Then the state at the first output mode becomes

$$|\phi\rangle \equiv \mathcal{N}_2\langle N_2|_3\langle N_3|_4\langle N_4|\Phi\rangle = \mathcal{N} \sum_{n=0}^{d-1} c_n^{(d)} \gamma_n |n\rangle \quad (3)$$

where $c_n^{(d)} = \langle n N_2 N_3 N_4 | \hat{U} | n_1 n_2 n_3 n \rangle$ depends on the beamsplitter transmittances $\mathbf{T} \equiv [t_1^2, t_2^2, t_3^2, t_4^2, t_5^2]$ and phase shifts $\xi \equiv [\xi_1, \xi_2, \xi_3, \xi_4, \xi_5]$. Then our task is to find $\mathbf{T}$ and ξ according to the desired state at the output in such a way that the fidelity of the output state to the desired state is maximized.

It is seen from Figs. 1 and 2 that eliminating the third modes at the input and output, and removing the components on the path from third input to the third output, the eight-port interferometer becomes the original QSD (six-port) when $|n_1\rangle_1 |n_2\rangle_2 = |1\rangle_1 |0\rangle_2$ and $|N_2\rangle_2 |N_4\rangle_4 = |1\rangle_2 |0\rangle_4$. In this case the optimized solution with the highest probability of successful truncation, that is to obtain the output state $|\phi^{(2)}\rangle \sim \gamma_0|0\rangle + \gamma_1|1\rangle$, becomes $\mathbf{T} = [t_1^2 = 1/2, t_4^2 = 1/2]$ and $\xi = [\xi_4 = \pi]$. In the same way Koniorczyk's QSD is obtained in the same six-port interferometer with $|n_1\rangle_1 |n_2\rangle_2 = |1\rangle_1 |1\rangle_2$ and $|N_2\rangle_2 |N_4\rangle_4 = |1\rangle_2 |1\rangle_4$. Then we find that there are four solutions for the successful truncation with the highest probability to obtain the state $|\phi^{(3)}\rangle \sim \gamma_0|0\rangle + \gamma_1|1\rangle + \gamma_2|2\rangle$. These solutions are $\mathbf{T}_1 = [t_1^2 = t_4^2 = (3 - \sqrt{3})/6]$, $\mathbf{T}_2 = [t_1^2 = t_4^2 = (3 + \sqrt{3})/6]$ if $\xi = [\xi_4 = 0]$, and $\mathbf{T}_3 = [t_1^2 = (3 - \sqrt{3})/6, t_4^2 = (3 + \sqrt{3})/6]$, $\mathbf{T}_4 = [t_1^2 (3 + \sqrt{3})/6, t_4^2 = (3 - \sqrt{3})/6]$ if $\xi = [\xi_4 = \pi]$. The first two solutions were given by Koniorczyk et. al, but the rest are found by us.

For the generalized QSD with the modified eight-port interferometer, we are more interested in the device to act as a QSD with a simple solution than the optimality of the solutions. We find that an input coherent state at the fourth-mode of the input can be truncated to give the output state $|\phi^{(4)}\rangle \sim \gamma_0|0\rangle + \gamma_1|1\rangle + \gamma_2|2\rangle + \gamma_3|3\rangle$ by inputting single photon states at $|n_1\rangle_1|n_2\rangle_2|n_3\rangle_3 = |1\rangle_1|1\rangle_2|1\rangle_3$ and by the conditional measurement $N_2 = N_3 = N_4 = 1$. In order for this to perform as a QSD we find a number of solutions. One simple solution is given by $\mathbf{T} = [1/3, 1/4, 1, 1/3, 1/2]$ with $\xi = [0, 0, 0, 0, \pi/2]$. Consequently, different output states with arbitrary coefficients can be obtained by proper choices of $\mathbf{T}$ and ξ provided that the total number of photons detected at the output detectors equal to the total number of input photons. For example, by inputting $|n_1\rangle_1|n_2\rangle_2|n_3\rangle_3 = |1\rangle_1|2\rangle_2|1\rangle_3$ and detecting $|N_2\rangle_2|N_3\rangle_3|N_4\rangle_4 = |1\rangle_2|2\rangle_3|1\rangle_4$, a truncated output state in the form $|\phi^{(5)}\rangle \sim \gamma_0|0\rangle + \gamma_1|1\rangle + \gamma_2|2\rangle + \gamma_3|3\rangle + \gamma_4|4\rangle$ can be obtained by choosing the BS and PS parameters as $\mathbf{T} = [0.305, 0.388, 1, 0.817, 0.184]$ with $\xi = [0, 0, 0, 0, \pi]$. For larger dimensional output states, it is difficult to obtain analytical solutions therefore solutions are found by numerical analysis with the condition that the fidelity of the output state to the desired one is the highest.

Quantum Punching Device: Here, we consider the cases where the output state obtained by truncating the input state $|\psi\rangle$ has some of its Fock states removed. Let us assume that $|k_1\rangle$ and $|k_2\rangle$ are removed, then the output state is written as

$$|\phi^{(d)}_{\text{punched } k_1,k_2,\ldots}\rangle = \mathcal{N} \sum_{\substack{n=0 \\ n\neq k_1,k_2}}^{d-1} \gamma_n|n\rangle. \tag{4}$$

We call this kind of process which opens holes in the Fock state expansion as the quantum punching device (QPD). We observed that by choosing proper BSs and PSs we can achieve this kind of state engieering using multiport interferometer. For example in the state $|\phi^{(4)}\rangle \sim \gamma_0|0\rangle + \gamma_1|1\rangle + \gamma_2|2\rangle + \gamma_3|3\rangle$, we can punch out (or remove) the $\gamma_2|2\rangle$ state by choosing $\mathbf{T} = [(7+\sqrt{21})/14, 1/3, 1, 1/2, (5-\sqrt{5})/10]$ with $\xi = [0, 0, 0, 0, 0]$. In the same way, we can obtain the state $|\phi^{(4)}_{\text{punched } 0}\rangle \sim \gamma_1|1\rangle + \gamma_2|2\rangle + \gamma_3|3\rangle$ with $\mathbf{T} = [(7+\sqrt{21})/14, 1/3, 1, 1/2, (2-\sqrt{2})/4]$ with $\xi = [0, 0, 0, 0, 0]$. We have observed that superpositions of any two Fock states

$$|\phi_{\text{punched k}\iota}\rangle \sim \gamma_k|k\rangle + \gamma_\iota|\iota\rangle \tag{5}$$

can be obtained as special cases of the truncation process, e.g., for $\xi = [0, 0, 0, 0, 0]$ and the transmittances given by

$$\begin{aligned} |\phi_{\text{punch. }02}\rangle \ &: \ \mathbf{T} = [1, 1/2, 1, 1, 1/2], \\ |\phi_{\text{punch. }13}\rangle \ &: \ \mathbf{T} = [\frac{1}{2}, \frac{3-\sqrt{3}}{3}, 1, \frac{3-\sqrt{3}}{3}, \frac{1}{2}]. \end{aligned} \tag{6}$$

It is interesting to see that one can synthesize two and three photon states in the $|\phi^{(4)}\rangle$ process by choosing

$$\begin{aligned} |2\rangle \ &: \ \mathbf{T} = [1, 1/2, 1/3, 1/2, 1], \\ |3\rangle \ &: \ \mathbf{T} = [1/2, 1/2, 1, 1/2, 1/2]. \end{aligned} \tag{7}$$

It must be noted we have given only some specific examples which guarantees the desired output state, and the solutions are usually not optimized.

CONCLUSION

We have shown that the original QSD scheme of Pegg-Phillips-Barnett can be generalized using multiport intereferometers. The original QSD scheme can be represented as a six-port interferometer. The multiport interferometer approach not only can help us to truncate a coherent state to obtain a superposition state up to an arbitrary Fock state but it also enables selective truncation of a given state and selective removal of Fock state components from it. As it was the case in the original QSD, the generalized one also produces the desired output state with very high fidelity when the input state to be truncated is a weak coherent state. A hard problem we face in this scheme is the optimization of the solutions to obtain the highest probability of truncation when d is high. In the present study, we did not focus in optimizing our solutions but in showing that the scheme is working as a truncation or punching device. Effects of imperfections (such as non-ideal photon counting and non-ideal single photon source) in the scheme are currently being investigated.

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