Selective truncations of an optical state using projection synthesis

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Selective truncation of the Fock-state expansion of an optical field can be achieved using projection synthesis. The process removes the predetermined Fock states from the input field by conditional measurement and teleportation. We present a scheme to perform projection synthesis based on multiport interferometry. This scheme can be used both as a generalized quantum scissors device that filters out Fock states with photon numbers higher than a predetermined value, and as a quantum punching device that selectively removes specific Fock states, making holes in the Fock-state expansion of the input field. © 2007 Optical Society of America

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1. INTRODUCTION

Recent theoretical and experimental works have prompted increasing interest in quantum-state engineering that uses linear optics. It has been shown that linear optics can be used for efficient quantum computations[1,2] and generation of arbitrary quantum states of traveling optical fields (see, for example, Refs. 3–15 and references therein). Such schemes are based on linear-optical elements, including beam splitters (BSs) and phase shifters (PSs), together with nonlinear elements such as single-photon sources and photodetectors. Parametric downconversion process is exploited to build a triggered single-photon source, and avalanche photodiodes are used as photon counters to discriminate between the absence and the presence of photons. Therefore, such schemes are experimentally realizable at the present level of optical technology.

In this paper, we study a linear-optical scheme for quantum-state engineering using projection synthesis.[8,9] Our main interest is to employ the scheme to perform the following transformation:

$$|\psi\rangle = \sum_{n=0}^{\infty} \gamma_n |n\rangle \rightarrow |\phi^{(d)}\rangle = \mathcal{N} \sum_{n=0}^{d-1} \gamma_n |n\rangle, \tag{1}$$

where the unknown input optical state $|\psi\rangle$ is truncated to obtain the state $|\phi^{(d)}\rangle$, which is a finite superposition of d states (for a review see Ref. 16). In Eq. (1), $\mathcal{N}$ is the normalization constant, which will be dropped from equations hereafter; thus, the sign $\sim$ will be used instead of equality to denote that the state should be normalized. This transformation is achieved by conditional measurement and teleportation. This process was originally described by Pegg, Phillips, and Barnett[8,9] to obtain a superposition state of $d=2$ of the form $|\phi^{(2)}\rangle \sim \gamma_0|0\rangle + \gamma_1|1\rangle$ by truncating a coherent state $|\psi\rangle = |\alpha\rangle$, and it was named as the *quantum scissors device* (QSD). Later we worked out a theoretical treatment of the QSD by proposing an experimentally realizable scheme and discussing how arbitrary superposition states of $d=2$ can be generated by this simple scheme.[17,18] The first experiment was performed by Babichev *et al.*[19] An extension of the original QSD scheme to $d=3$ was proposed Koniorczyk *et al.*[20] by a simple modification of the original QSD scheme. The original QSD scheme is interesting because it finds its direct application as a basic element of a single-rail version of the linear-optical quantum computer. Moreover, it is not only a truncation scheme but also a communication scheme for superposition states of arbitrary d.

The drawback of the original QSD scheme is that it enables generation of truncated states up to $d=3$. In this paper, we extend the results of Ref. 21 to describe an application of a modified version of the multiport Mach–Zehnder interferometer in the configuration of Zeilinger *et al.*,[4,5] which has been experimentally demonstrated.[6,7] The important difference between the original multiport interferometer and the modified version discussed here is the elimination of the apex BS so that the direct path from the input field to the output field is eliminated. This is crucial for the truncation scheme, as we want the process to be realized via teleportation.

In the following section, we will introduce the generalized QSD scheme based on the multiport interferometer and give some examples of the possible truncated states. Then we will discuss how the same scheme can be used as a quantum punching device, which selectively eliminates some Fock states from the original superposition state and makes holes in the Fock-state expansion provided the proper choices are made regarding conditional measurement and input states.

2. MULTIPORT INTERFEROMETER AS QUANTUM SCISSORS DEVICE

A schematic diagram of the eight-port Mach–Zehnder interferometer in the configuration of Zeilinger *et al.*[4] and the generalized QSD is given in Fig. 1. As a special case, the original Pegg–Phillips–Barnett scheme of QSD can be considered as a six-port interferometer presented in Fig. 2. The beam splitter shown with the dotted lines corresponds to the apex BS that is to be removed from the interferometer in order to obtain the generalized QSD scheme. If we define the N-mode input state as $|\Psi\rangle$, then the N-mode output state will be $|\Phi\rangle=\hat{U}|\Psi\rangle$, where $\hat{U}$ is the

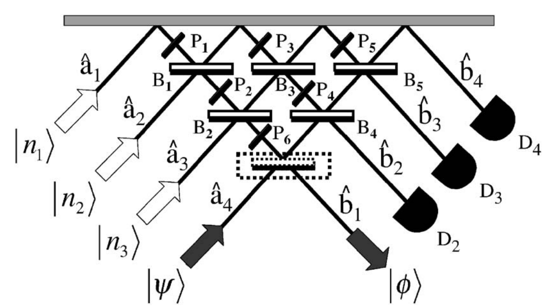

Fig. 1. Generalized eight-port QSD. Notation: $|\psi\rangle$ is the input state to be truncated, usually a coherent state $|\alpha\rangle$; $|n_j\rangle$—input Fock states; $|\phi\rangle$—output state, selectively truncated or punched; D_j—photon counters; B_j—beam splitters; P_j—phase shifters; and $\hat{a}_j$ and $\hat{b}_j$—input and output annihilation operators, respectively. The beams reflected from the white surface of the beam splitters are π phase shifted, while the reflections from the black surface are without any phase shift. The beam splitter drawn with a dotted line and bounded by the dotted box is the one to be removed in order to obtain the QSD from the conventional multiport interferometer.

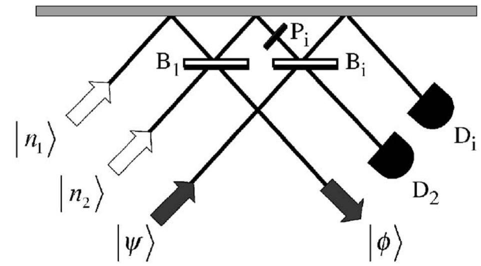

Fig. 2. Representation of the original Pegg–Phillips–Barnett scheme of QSD as a six-port interferometer scheme. Notation is the same as in Fig. 1, where $i=4$ (or equivalently $i=3$).

unitary operator describing the evolution of the input state in the interferometer. Denoting the annihilation operators at the input and output ports as column vectors $\hat{\mathbf{a}}\equiv[\hat{a}_1;\hat{a}_2;\ldots;\hat{a}_N]$ and $\hat{\mathbf{b}}\equiv[\hat{b}_1;\hat{b}_2;\ldots;\hat{b}_N]$, respectively, we obtain $\hat{\mathbf{b}}=\hat{U}^{\dagger}\hat{\mathbf{a}}\hat{U}=S\hat{\mathbf{a}}$, where $S=P_6B_5P_5B_4P_4B_3P_3B_2P_2B_1P_1$ is the scattering matrix obtained by multiplying the scattering matrices of the BSs, B_i, and PSs, P_i, used in the scheme from the input to the output. We assume B_i $(i=1,\ldots,5)$ to be described by a real 2×2 matrix $[t_i,r_i;-r_i,t_i]$ embedded in a 4×4 matrix, where t_i^2 and $r_i^2=1-t_i^2$ are the BS transmittance and reflectance, respectively. Internal phase shifts of BSs can formally be included by using external PSs described by parameters ξ_i. For simplicity, we analyze the system without P_6, i.e., assuming $\xi_6=0$.

Now, considering that at the input port we have the Fock state $|\Psi\rangle=|n_1,\ldots,n_N\rangle\equiv|\mathbf{n}\rangle$, the output state is found as

$$|\Phi\rangle=\hat{U}|\mathbf{n}\rangle=\frac{1}{\sqrt{n_1!\cdots n_N!}}\sum_{\mathbf{j}=1}^{N}\prod_{l=1}^{\nu}S_{j_lx_l}\hat{a}_{j_l}^{\dagger}|\mathbf{0}\rangle, \tag{2}$$

where $S_{j_lx_l}$ are the elements of the unitary scattering matrix S, $\nu=\Sigma_i n_i$ is the total number of photons, and $\Sigma_{\mathbf{j}}$ stands for the multiple sum over $j_1,j_2,\ldots,j_\nu$. Moreover, $x_l=j$ for $\Sigma_{i=1}^{j-1}n_i<l\leq\Sigma_{i=1}^{j}n_i$ and $j=1,\ldots,N$.

3. SELECTIVE-STATE TRUNCATIONS

A. Quantum Scissors Device

In a truncation scheme, we are interested in obtaining a superposition state by truncating the input optical state, which is usually a coherent state. Therefore, in the generalized QSD scheme, based on the multiport interferometer shown in Fig. 1, we consider the state $|\psi\rangle$ as one of the inputs. In that case, for the eight-port interferometer we can write the total input state as $|\Psi\rangle=|n_1\rangle_1|n_2\rangle_2|n_3\rangle_3|\psi\rangle_4$. Now assume that the detectors at the output ports detect N_2, N_3, and N_4 photons whose sum is the total number of photons input into the interferometer and satisfies the relation

$$N_2+N_3+N_4=n_1+n_2+n_3=d-1. \tag{3}$$

This means that we project the total output state $|\Phi\rangle_{1,2,3,4}$ onto the detected states $|N_2\rangle_2|N_3\rangle_3|N_4\rangle_4$. Then the state at the first output mode becomes

$$|\phi\rangle \sim {}_2\langle N_2|_3\langle N_3|_4\langle N_4|\Phi\rangle = \sum_{n=0}^{d-1} c_n^{(d)}\gamma_n|n\rangle, \tag{4}$$

where $c_n^{(d)}=\langle n,N_2,N_3,N_4|\hat{U}|n_1,n_2,n_3,n\rangle$ depends on the beam-splitter transmittances $\mathbf{T}\equiv[t_1^2,t_2^2,t_3^2,t_4^2,t_5^2]$ and phase shifts $\boldsymbol{\xi}\equiv[\xi_1,\xi_2,\xi_3,\xi_4,\xi_5]$. Then our task is to find $\mathbf{T}$ and $\boldsymbol{\xi}$ according to the desired state at the output in such a way that the fidelity of the output state to the desired state is maximized.

It is seen from Figs. 1 and 2 that by eliminating the third modes at the input and output and removing the components on the path from the third input to the third output, the eight-port interferometer becomes the original six-port QSD when $|n_1\rangle_1|n_2\rangle_2=|1\rangle_1|0\rangle_2$ and $|N_2\rangle_2|N_4\rangle_4=|1\rangle_2|0\rangle_4$. In this case, the optimized solution with the highest probability of successful truncation, that is, corresponding to the output state

$$|\phi^{(2)}\rangle \sim \gamma_0|0\rangle + \gamma_1|1\rangle, \tag{5}$$

becomes $\mathbf{T}=[t_1^2=1/2,t_4^2=1/2]$ and $\boldsymbol{\xi}=[\xi_4=\pi]$. In the same way the QSD of Koniorczyk *et al.*[20] is obtained in this six-port interferometer with $|n_1\rangle_1|n_2\rangle_2=|1\rangle_1|1\rangle_2$ and $|N_2\rangle_2|N_4\rangle_4=|1\rangle_2|1\rangle_4$. Then we find that there are four solutions for the successful truncation with the highest probability to obtain the state

$$|\phi^{(3)}\rangle \sim \gamma_0|0\rangle + \gamma_1|1\rangle + \gamma_2|2\rangle. \tag{6}$$

These solutions are $\mathbf{T}_1=[t_1^2=t_4^2=(3-\sqrt{3})/6]$; $\mathbf{T}_2=[t_1^2=t_4^2=(3+\sqrt{3})/6]$ if $\boldsymbol{\xi}=[\xi_4=0]$; $\mathbf{T}_3=[t_1^2=(3-\sqrt{3})/6,t_4^2=(3+\sqrt{3})/6]$; and $\mathbf{T}_4=[t_1^2=(3+\sqrt{3})/6,t_4^2=(3-\sqrt{3})/6]$ if $\boldsymbol{\xi}=[\xi_4=\pi]$. The first two solutions were given by Koniorczyk *et al.*,[20] but the rest have been found by us.

For the generalized QSD with the modified eight-port interferometer, we are more interested in the device in its capacity to act as a QSD with a simple solution rather than in the optimality of the solutions. We find that an input coherent state at the fourth mode of the input can be truncated to give the output state

$$|\phi^{(4)}\rangle \sim \gamma_0|0\rangle + \gamma_1|1\rangle + \gamma_2|2\rangle + \gamma_3|3\rangle \tag{7}$$

by inputting single-photon states at $|n_1\rangle_1|n_2\rangle_2|n_3\rangle_3=|1\rangle_1|1\rangle_2|1\rangle_3$ and by the conditional measurement $N_2=N_3=N_4=1$. We find a number of solutions for transmittances and phase shifts in the QSD, for which the input state is truncated to form relation (7). One simple solution is given by $\mathbf{T}=[1/3,1/4,1,1/3,1/2]$ with $\boldsymbol{\xi}=[0,0,0,0,\pi/2]$.

Consequently, various output states with desired coefficients can be obtained by making the proper choices for $\mathbf{T}$ and $\boldsymbol{\xi}$ provided that the total number of photons detected at the output detectors is equal to the total number of input photons. For example, by inputting $|n_1\rangle_1|n_2\rangle_2|n_3\rangle_3=|1\rangle_1|2\rangle_2|1\rangle_3$ and detecting $|N_2\rangle_2|N_3\rangle_3|N_4\rangle_4=|1\rangle_2|2\rangle_3|1\rangle_4$, we can obtain a truncated output state in the form

$$|\phi^{(5)}\rangle \sim \gamma_0|0\rangle + \gamma_1|1\rangle + \gamma_2|2\rangle + \gamma_3|3\rangle + \gamma_4|4\rangle \tag{8}$$

by choosing the BS and PS parameters as $\mathbf{T}=[0.305,0.388,1,0.817,0.184]$ with $\boldsymbol{\xi}=[0,0,0,\pi,0]$. For larger-dimensional output states, it is difficult to obtain analytical solutions; therefore, solutions are found by numerical analysis on condition that the fidelity of the output state to the desired (ideally truncated) state is equal to one.

B. Quantum Punching Device

Here we consider cases where the output state obtained by truncating the input state $|\psi\rangle$ has some of its Fock states removed. Let us assume that $|r_1\rangle,|r_2\rangle,\ldots$ are removed. Then the output state is written as

$$|\phi_{\text{punch}}^{(d)}\rangle \sim \sum_{\substack{n=0\\ n\neq r_1,r_2,\ldots}}^{d-1} \gamma_n|n\rangle. \tag{9}$$

This process can be referred to as *hole burning*[22–24] or *quantum punching* in the Fock-state expansion of a given state of light. Thus we refer to our system as a *quantum punching device* (QPD).

We have applied the following procedure to find a desired selective superposition, given by relation (9). In particular, the procedure can also be applied for the standard truncation without punching. The QPD should perform a desired selective truncation for any input state $|\psi\rangle$ to be engineered for given auxiliary input states $|n_1\rangle_1$, $|n_2\rangle_2$, and $|n_3\rangle_3$ and for measured states $|N_2\rangle_2$, $|N_3\rangle_3$, and $|N_4\rangle_4$ that satisfy Eq. (3). The amplitudes should fulfill the condition $c_n^{(d)}=\text{const}>0$ if $n\neq r_i$ for $i=1,2,\ldots$ and vanish for the other n. So the problem is to find such transmittances $\mathbf{T}$ and phase shifts $\boldsymbol{\xi}$ for which an auxiliary function

$$\Delta \equiv \sum_{n\neq r_i} |c_n^{(d)} - c_{n'}^{(d)}| + \sum_{r_i} |c_{r_i}^{(d)}| \tag{10}$$

vanishes, where n' is one of $n\neq r_i$. We have performed numerical minimalization (specifically, based on a simplex search method) of Δ for randomly chosen $\mathbf{T}$ and $\boldsymbol{\xi}$ to get zero up to double precision.

The total number N_d of selective truncations of a given state $|\phi^{(d)}\rangle$ for arbitrary d is given by

$$N_d = \sum_{n=1}^{d} \binom{d}{n} = 2^d - 1, \tag{11}$$

where $\binom{d}{n}$ stands for binomial coefficient. Note that trivial cases like $|0\rangle$ and state $|\phi^{(d)}\rangle$ are also taken into account in Eq. (11). By choosing proper BSs and PSs, we can achieve this kind of state engineering using the multiport interferometer. To demonstrate explicitly the capabilities of our scheme, let us analyze all $N_d=15$ selective truncations in the state $|\phi^{(4)}\rangle$, given by relation (7). We can punch out (or remove) the state $|0\rangle$, denoted by a filled circle, to get

$$|\phi_{\text{punch}\bullet 123}^{(4)}\rangle \sim \gamma_1|1\rangle + \gamma_2|2\rangle + \gamma_3|3\rangle, \tag{12}$$

by choosing $\mathbf{T}=[(7+\sqrt{21})/14,1/3,1,1/2,(2-\sqrt{2})/4]$. Hereafter, we assume the input Fock states $|n_i\rangle_i=|1\rangle_i (i=1,2,3)$, all measurement results equal to one, and all phase shifts equal to zero, i.e., $\boldsymbol{\xi}=\mathbf{0}$ except generation of state $|3\rangle$.

Analogously, we can punch out state $|1\rangle$ to get

$$|\phi^{(4)}_{\text{punch}0\bullet 23}\rangle \sim \gamma_0|0\rangle + \gamma_2|2\rangle + \gamma_3|3\rangle, \quad (13)$$

by setting $2\mathbf{T}=[1-3\sqrt{5/173},1,2,1/3,1+5\sqrt{3/203}]$, and punch out $|2\rangle$ to get

$$|\phi^{(4)}_{\text{punch}01\bullet 3}\rangle \sim \gamma_0|0\rangle + \gamma_1|1\rangle + \gamma_3|3\rangle, \quad (14)$$

by choosing $\mathbf{T}=[(7+\sqrt{21})/14,1/3,1,1/2,(5-\sqrt{5})/10]$. Note that $|\phi^{(4)}_{\text{punch }012\bullet}\rangle=|\phi^{(3)}\rangle$ and is given by relation (6).

Moreover, all six superpositions of two Fock states can be obtained in the scheme, e.g., for $\boldsymbol{\xi}=\mathbf{0}$. In a simplified system with the BS B_3 removed ($t_3=1$), one can get the following states:

$$|\phi^{(4)}_{\text{punch}0\bullet 2\bullet}\rangle \text{ for } \mathbf{T}=\left[1,\tfrac{1}{2},1,1,\tfrac{1}{2}\right],$$

$$|\phi^{(4)}_{\text{punch}\bullet 1\bullet 3}\rangle \text{ for } \mathbf{T}=\left[\tfrac{1}{2},t^2,1,t^2,\tfrac{1}{2}\right],$$

$$|\phi^{(4)}_{\text{punch}0\bullet\bullet 3}\rangle \text{ for } \mathbf{T}=\left[(t')^2,\tfrac{1}{2},1,\tfrac{1}{6},(t'')^2\right], \quad (15)$$

where $t^2=(3-\sqrt{3})/3$, $(t')^2=(1-\sqrt{5/133})/2$, and $(t'')^2=(1+3\sqrt{3/155})/2$, and

$$|\phi^{(4)}_{\text{punch}\bullet\bullet 23}\rangle \text{ for } \mathbf{T}=\left[t^2,\tfrac{1}{2},1,\tfrac{1}{6},(t')^2\right], \quad (16)$$

assuming $t^2=(1-\sqrt{5/37})/2$ and $(t')^2=(1+\sqrt{3/35})/2$. One can also generate state

$$|\phi^{(4)}_{\text{punch}\bullet 12\bullet}\rangle \text{ for } \mathbf{T}=\left[t^2,\tfrac{8}{9},\tfrac{1}{2},t^2,1\right], \quad (17)$$

with $t^2=1/2+1/\sqrt{5}$. We note that this state can also be obtained for a system with $t_3=1$, assuming that one of the inputs is in vacuum state and that no photons are measured in one of the outputs. The remaining sixth state is trivial as corresponds to $|\phi^{(4)}_{\text{punch}01\bullet\bullet}\rangle=|\phi^{(2)}\rangle$, given by relation (5).

It is interesting to see that one can synthesize two- and three-photon Fock states in the $|\phi^{(4)}\rangle$ process by choosing $\mathbf{T}=[1,1/2,1/3,1/2,1]$ and $\mathbf{T}=[1/2,1/2,1,1/2,1/2]$, respectively, and by assuming $\boldsymbol{\xi}=\mathbf{0}$ except $\xi_5=\pi/2$ in the latter case.

It must be noted that we have given only some specific examples, which guarantee the desired output state, but the solutions are usually not optimized for the success probability.

We have tested our scheme for selective truncations up to $d=6$. We have found solutions for many but not all possible $N_d=63$ superpositions. Nevertheless, the system is easily scalable, so by increasing the number of BSs, auxiliary input states, and measured output states, we can in principle generate an arbitrary superposition state via teleportation.

4. CONCLUSION

We have shown that the original Pegg–Phillips–Barnett scheme of QSD can be generalized by using multiport Mach–Zehnder interferometers in the configuration of Zeilinger *et al.*[4] The original QSD scheme can be represented as a six-port interferometer. The multiport interferometer approach can help us not only to truncate a coherent state in order to obtain a superposition state up to an arbitrary Fock state but also to enable selective truncation of a given state and selective removal of Fock-state components from it—a process referred to as quantum punching or hole burning in the Fock space of optical fields. It should be noted that several schemes for hole burning have already been proposed based on conditional measurements on linear (Refs. 22 and 23 and references therein) and nonlinear[24] systems. Nevertheless, the present scheme is the first that enables state truncation and hole burning simultaneously. Moreover, contrary to former schemes, the process is achieved via teleportation.

As was the case in the original QSD, the generalized one also produces the desired output state with very high fidelity when the input state to be truncated is a weak coherent state. A difficult problem we face in this scheme entails optimization of the solutions to obtain the highest probability of truncation when d is high. In the present study, we did not focus on optimization of our solutions but on showing that the scheme works as a truncation or punching device. The effects of imperfections (such as nonideal photon counting and nonideal single-photon source) in the scheme are currently being investigated.

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REFERENCES

1. E. Knill, R. Laflamme, and G. J. Milburn, “A scheme for efficient quantum computation with linear optics,” Nature **409**, 46–52 (2001).
2. R. Raussendorf and H. J. Briegel, “A one-way quantum computer,” Phys. Rev. Lett. **86**, 5188–5191 (2001).
3. Special issue on *Quantum State Preparation and Measurement*, J. Mod. Opt. **44** (1997).
4. A. Zeilinger, H. J. Bernstein, D. M. Greenberger, A. M. Horne, and M. Żukowski, “Controlling entanglement in quantum optics,” in *Quantum Control and Measurement*, H. Ezawa and Y. Murayama, eds. (North Holland, 1993) pp. 9–22.
5. M. Reck, A. Zeilinger, H. J. Bernstein, and P. Bertani, “Experimental realization of any discrete unitary operator,” Phys. Rev. Lett. **73**, 58–61 (1994).
6. C. Mattle, M. Michler, H. Weinfurter, A. Zeilinger, and M. Żukowski, “Non-classical statistics at multiport beam splitters,” Appl. Phys. B: Lasers Opt. **60**, S111–S117 (1995).
7. M. Reck, “Quantum interferometry with multiports: entangled photons in optical fibers,” Ph.D. thesis (University of Innsbruck, 1996).
8. D. T. Pegg, L. S. Phillips, andS. M. Barnett, “Optical state truncation by projection synthesis,” Phys. Rev. Lett. **81**, 1604–1606 (1998).
9. S. M. Barnett and D. T. Pegg, “Optical state truncation,” Phys. Rev. A **60**, 4965–4973 (1999).
10. M. Dakna, J. Clausen, L. Knöll, and D.-G. Welsch, “Generation of arbitrary quantum states of traveling fields,” Phys. Rev. A **59**, 1658–1661 (1999).
11. C. J. Villas-Bôas, Y. Guimarães, M. H. Y. Moussa, and B. Baseia, “Recurrence formula for generalized optical state

truncation by projection synthesis," Phys. Rev. A **63**, 055801 (2001).

12. K. J. Resch, J. S. Lundeen, and A. M. Steinberg, "Quantum state preparation and conditional coherence," Phys. Rev. A **88**, 113601 (2002).
13. A. I. Lvovsky and J. Mlynek, "Quantum-optical catalysis: generating nonclassical states of light by means of linear optics," Phys. Rev. Lett. **88**, 250401 (2002).
14. J. Fiurášek, S. Massar, and N. J. Cerf, "Conditional generation of arbitrary multimode entangled states of light with linear optics," Phys. Rev. A **68**, 042325 (2003).
15. J. Clausen, L. Knöll, and D.-G. Welsch, "Realization of multimode operators with passive linear optics and photodetection," Phys. Rev. A **68**, 043822 (2003).
16. A. Miranowicz, W. Leoński, and N. Imoto, "Quantum-optical states in finite-dimensional Hilbert space. I. General formalism," Adv. Chem. Phys. **119**, 155–193 (2001).
17. Ş. K. Özdemir, A. Miranowicz, M. Koashi, and N. Imoto, "Quantum scissors device for optical state truncation: a proposal for practical realization," Phys. Rev. A **64**, 063818 (2001).
18. Ş. K. Özdemir, A. Miranowicz, M. Koashi, and N. Imoto, "Pulse-mode quantum projection synthesis: effects of mode mismatch on optical state truncation and preparation," Phys. Rev. A **66**, 053809 (2002).
19. S. A. Babichev, J. Ries, and A. I. Lvovsky, "Quantum scissors: teleportation of single-mode optical states by means of a nonlocal single photon," Europhys. Lett. **64**, 1–7 (2003).
20. M. Koniorczyk, Z. Kurucz, A. Gábris, and J. Janszky, "General optical state truncation and its teleportation," Phys. Rev. A **62**, 013802 (2000).
21. A. Miranowicz, "Optical-state truncation and teleportation of qudits by conditional eight-port interferometry," J. Opt. B **7**, 142–150 (2005).
22. B. Baseia, M. H. Y. Moussa, and V. S. Bagnato, "Hole burning in Fock space," Phys. Lett. A **240**, 277–281 (1998).
23. B. M. Escher, A. T. Avelar, T. M. da Rocha Filho, and B. Baseia, "Controlled hole burning in the Fock space via conditional measurements on beam splitters," Phys. Rev. A **70**, 025801 (2004).
24. C. C. Gerry and A. Benmoussa, "Hole burning in the Fock space of optical fields," Phys. Lett. A **303**, 30–36 (2002).