Priority Choice Experimental Two-qubit Tomography: Measuring One by One All Elements of Density Matrices: Supplementary Material

Karol Bartkiewicz, $^{1,\,2,\,*}$ Antonín Černoch, 3 Karel Lemr, 2 and Adam Miranowicz $^{4,\,1}$

¹Faculty of Physics, Adam Mickiewicz University, PL-61-614 Poznań, Poland

 2RCPTM , Joint Laboratory of Optics of Palacký University and

Institute of Physics of Academy of Sciences of the Czech Republic,

17. listopadu 12, 772 07 Olomouc, Czech Republic

³Institute of Physics of Academy of Science of the Czech Republic,

Joint Laboratory of Optics of Palacký University and Institute of Physics of Academy of Sciences of the Czech Republic,

17. listopadu 50A, 77207 Olomouc, Czech Republic ⁴CEMS, RIKEN, 351-0198 Wako-shi, Japan

(Dated: October 22, 2015)

Here we show explicitly all the density matrices discussed in the Letter, which are reconstructed with the optimal tomographic protocol and those based on: (i) mutually unbiased bases, (ii) the James-Kwiat-Munro-White projectors, (iii) the tensor products of the Pauli operators, and (iv) the standard separable basis corresponding to all the eigenvectors of the Pauli operators. We also present the coefficient matrices, observation vectors corresponding to coincidence counts, the estimated variances for the observations, and the error radii for each reconstructed matrix. Finally, we compare the reconstructed matrices graphically, where we show the relative trace distances between the reconstructed states and they error radii.

* bark@amu.edu.pl

I. RECONSTRUCTED DENSITY MATRICES

The 17 density matrices are reconstructed by solving linear inversion problem for four tomographies. We have prepared 17 different states of high purity, which approximately correspond to:

$$\begin{aligned} |\psi_{1}\rangle &= (|HH\rangle - |VV\rangle)/\sqrt{2}, & |\psi_{2}\rangle &= (|HH\rangle + |VV\rangle)/\sqrt{2}, & |\psi_{3}\rangle &= (|HH\rangle - i|VV\rangle)/\sqrt{2}, \\ |\psi_{4}\rangle &= (|DR\rangle - i|AL\rangle)/\sqrt{2}, & |\psi_{5}\rangle &= (|HV\rangle + i|VH\rangle)/\sqrt{2}, & |\psi_{6}\rangle &= (|HV\rangle + |VH\rangle)/\sqrt{2}, \\ |\psi_{7}\rangle &= |HV\rangle, & |\psi_{8}\rangle &= (|HH\rangle + i|VV\rangle)/\sqrt{2}, & |\psi_{9}\rangle &= (|HV\rangle - |VH\rangle)/\sqrt{2}, \\ |\psi_{10}\rangle &= (|HV\rangle - i|VH\rangle)/\sqrt{2}, & |\psi_{11}\rangle &= (|DL\rangle + i|AR\rangle)/\sqrt{2}, & |\psi_{12}\rangle &= (|DL\rangle - i|AR\rangle)/\sqrt{2}, \\ |\psi_{13}\rangle &= |e_{1a}e_{1b}\rangle, & |\psi_{14}\rangle &= |e_{2a}e_{2b}\rangle, & |\psi_{15}\rangle &= 0.79|HV\rangle - 0.61|VH\rangle, \\ |\psi_{16}\rangle &= 0.50|HV\rangle - 0.87|VH\rangle, & |\psi_{17}\rangle &= 0.35|HV\rangle - 0.94|VH\rangle; \end{aligned}$$

where $|e_{1a}\rangle = (-0.6556 + 0.6248i)|H\rangle + 0.4241|V\rangle$, $|e_{1b}\rangle = (-0.1415 - 0.7165i)|H\rangle + 0.6831|V\rangle$, $|e_{2a}\rangle = (-0.9608 + 0.2091i)|H\rangle + 0.1822|V\rangle$, and $|e_{2b}\rangle = (0.2613 + 0.7338i)|H\rangle + 0.6271|V\rangle$ are the single photon-elliptic polarization states. In the subsequent sections, the states are numbered accordingly. Note that the states selected for the main text of our paper are defined as $|\phi_1\rangle \equiv |\psi_4\rangle$, $|\phi_2\rangle \equiv |\psi_7\rangle$, $|\phi_3\rangle \equiv |\psi_9\rangle$, and $|\phi_4\rangle \equiv |\psi_{14}\rangle$. We mark the data relevant to a particular tomography as follows: index O for the optimal tomography; S for the standard 36-state tomography; J for the James-Kwiat-Munro-White (JKMW) protocol; M for the MUB-based tomography; P for the Pauli-matrices-based tomography.

A. Standard 36-state tomography

$ \rho_{S,1} = $	$\begin{bmatrix} 0.4922 \\ 0.0020 - 0.0156i \\ -0.0042 - 0.0354i \\ -0.4607 + 0.0750i \end{bmatrix}$	$\begin{array}{c} 0.0020 + 0.0156i \\ 0.0047 \\ -0.0054 - 0.0228i \\ 0.0255 + 0.0002i \end{array}$	$\begin{array}{c} -0.0042 + 0.0354i \\ -0.0054 + 0.0228i \\ 0.0136 \\ 0.0184 - 0.0656i \end{array}$	$ \begin{bmatrix} -0.4607 - 0.0750i \\ 0.0255 - 0.0002i \\ 0.0184 + 0.0656i \\ 0.4895 \end{bmatrix} $
$\rho_{S,2} =$	$\begin{bmatrix} 0.4870 \\ 0.0029 + 0.0358i \\ 0.0237 + 0.0103i \\ 0.4723 - 0.0515i \end{bmatrix}$	$\begin{array}{r} 0.0029 - 0.0358i \\ 0.0085 \\ -0.0000 + 0.0219i \\ 0.0244 - 0.0413i \end{array}$	$\begin{array}{c} 0.0237 - 0.0103i \\ -0.0000 - 0.0219i \\ 0.0038 \\ -0.0052 + 0.0378i \end{array}$	$ \begin{bmatrix} 0.4723 + 0.0515i \\ 0.0244 + 0.0413i \\ -0.0052 - 0.0378i \\ 0.5007 \end{bmatrix} $
$ \rho_{S,3} =$	$\begin{bmatrix} 0.5363 \\ 0.0744 + 0.0611i \\ 0.0830 + 0.0243i \\ -0.0027 - 0.4636i \end{bmatrix}$	$\begin{array}{c} 0.0744 - 0.0611i\\ 0.0206\\ 0.0522 + 0.0036i\\ -0.0513 - 0.0602i \end{array}$	$\begin{array}{c} 0.0830 - 0.0243i \\ 0.0522 - 0.0036i \\ 0.0002 \\ 0.0005 - 0.0402i \end{array}$	$ \begin{bmatrix} -0.0027 + 0.4636i \\ -0.0513 + 0.0602i \\ 0.0005 + 0.0402i \\ 0.4429 \end{bmatrix} $
$ \rho_{S,4} = $	$\begin{bmatrix} 0.3005\\ 0.2633 - 0.0250i\\ 0.0456 + 0.2223i\\ -0.0648 - 0.2710i \end{bmatrix}$	$\begin{array}{c} 0.2633 + 0.0250i \\ 0.2482 \\ 0.0417 + 0.1841i \\ -0.0334 - 0.2402i \end{array}$	$\begin{array}{c} 0.0456 - 0.2223i\\ 0.0417 - 0.1841i\\ 0.1270\\ -0.2145 + 0.0463i\end{array}$	$\begin{array}{c} -0.0648 + 0.2710i \\ -0.0334 + 0.2402i \\ -0.2145 - 0.0463i \\ 0.3244 \end{array} \right]$
$ \rho_{S,5} =$	$\begin{bmatrix} 0.0135 \\ -0.0859 - 0.0013i \\ 0.0334 - 0.0685i \\ 0.0159 - 0.0018i \end{bmatrix}$	$\begin{array}{c} -0.0859 + 0.0013i\\ 0.5111\\ 0.0397 + 0.4647i\\ -0.0002 + 0.0441i\end{array}$	$\begin{array}{c} 0.0334 + 0.0685i\\ 0.0397 - 0.4647i\\ 0.4697\\ 0.0078 + 0.0236i\end{array}$	$ \begin{bmatrix} 0.0159 + 0.0018i \\ -0.0002 - 0.0441i \\ 0.0078 - 0.0236i \\ 0.0056 \end{bmatrix} $
$ \rho_{S,6} = $	$\begin{bmatrix} 0.0146 \\ 0.0450 - 0.0866i \\ 0.0849 - 0.0819i \\ -0.0046 - 0.0341i \end{bmatrix}$	$\begin{array}{c} 0.0450 + 0.0866i \\ 0.4606 \\ 0.4608 + 0.0160i \\ -0.0222 + 0.0282i \end{array}$	$\begin{array}{c} 0.0849 + 0.0819i \\ 0.4608 - 0.0160i \\ 0.5177 \\ -0.0526 + 0.0125i \end{array}$	$ \begin{bmatrix} -0.0046 + 0.0341i \\ -0.0222 - 0.0282i \\ -0.0526 - 0.0125i \\ 0.0072 \end{bmatrix} $

$ \rho_{S,7} = $	$\begin{bmatrix} 0.0005\\ 0.0578 + 0.0482i\\ 0.0064 - 0.0028i\\ -0.0088 + 0.0005i \end{bmatrix}$	$\begin{array}{c} 0.0578 - 0.0482i \\ 0.9915 \\ 0.0290 - 0.0678i \\ -0.0436 + 0.1064i \end{array}$	$\begin{array}{c} 0.0064 + 0.0028i \\ 0.0290 + 0.0678i \\ 0.0049 \\ -0.0043 + 0.0022i \end{array}$	$\begin{array}{c} -0.0088 - 0.0005i \\ -0.0436 - 0.1064i \\ -0.0043 - 0.0022i \\ 0.0032 \end{array} \right]$
$ \rho_{S,8} = $	$\begin{bmatrix} 0.5609 \\ -0.0543 - 0.0315i \\ 0.0357 + 0.0364i \\ 0.0027 + 0.4704i \end{bmatrix}$	$\begin{array}{c} -0.0543 + 0.0315i \\ 0.0067 \\ -0.0470 - 0.0079i \\ -0.0016 - 0.0511i \end{array}$	$\begin{array}{c} 0.0357-0.0364i\\ -0.0470+0.0079i\\ -0.0091\\ 0.0065+0.0126i\end{array}$	$ \begin{bmatrix} 0.0027 - 0.4704i \\ -0.0016 + 0.0511i \\ 0.0065 - 0.0126i \\ 0.4416 \end{bmatrix} $
$ \rho_{S,9} =$	$\begin{bmatrix} -0.0208 \\ -0.0024 + 0.0670i \\ -0.0036 - 0.0319i \\ -0.0224 + 0.0403i \end{bmatrix}$	$\begin{array}{r} -0.0024 - 0.0670i \\ 0.5767 \\ -0.4584 + 0.0718i \\ -0.0076 + 0.0478i \end{array}$	$\begin{array}{c} -0.0036 + 0.0319i \\ -0.4584 - 0.0718i \\ 0.4334 \\ 0.0134 - 0.0045i \end{array}$	$\begin{array}{c} -0.0224 - 0.0403i \\ -0.0076 - 0.0478i \\ 0.0134 + 0.0045i \\ 0.0107 \end{array} \right]$
$ \rho_{S,10} = $	$= \begin{bmatrix} -0.0118\\ 0.0243 + 0.0103i\\ 0.0134 - 0.0063i\\ -0.0090 + 0.0064i \end{bmatrix}$	$\begin{array}{c} 0.0243 - 0.0103i \\ 0.5080 \\ 0.0499 - 0.4684i \\ -0.0174 + 0.0050i \end{array}$	$\begin{array}{c} 0.0134 + 0.0063i \\ 0.0499 + 0.4684i \\ 0.4801 \\ 0.0302 - 0.0537i \end{array}$	$ \begin{bmatrix} -0.0090 - 0.0064i \\ -0.0174 - 0.0050i \\ 0.0302 + 0.0537i \\ 0.0237 \end{bmatrix} $
$ \rho_{S,11} = $	$\begin{bmatrix} 0.2826\\ 0.2502 - 0.0153i\\ -0.0157 - 0.2358i\\ 0.0053 + 0.2593i \end{bmatrix}$	$\begin{array}{c} 0.2502 + 0.0153i \\ 0.2221 \\ -0.0295 - 0.2241i \\ 0.0222 + 0.2349i \end{array}$	$\begin{array}{c} -0.0157 + 0.2358 i \\ -0.0295 + 0.2241 i \\ 0.2718 \\ -0.2432 - 0.0003 i \end{array}$	$\begin{array}{c} 0.0053 - 0.2593i \\ 0.0222 - 0.2349i \\ -0.2432 + 0.0003i \\ 0.2235 \end{array}$
$ \rho_{S,12} = $	$= \begin{bmatrix} 0.2100 \\ -0.2611 + 0.0385i \\ -0.0419 + 0.2516i \\ 0.0345 + 0.2156i \end{bmatrix}$	$\begin{array}{c} -0.2611 - 0.0385i \\ 0.2815 \\ 0.0452 - 0.2460i \\ 0.0115 - 0.2405i \end{array}$	$\begin{array}{c} -0.0419 - 0.2516a \\ 0.0452 + 0.2460i \\ 0.2312 \\ 0.2259 - 0.0926i \end{array}$	$ \begin{bmatrix} 0.0345 - 0.2156i \\ 0.0115 + 0.2405i \\ 0.2259 + 0.0926i \\ 0.2772 \end{bmatrix} $
$ \rho_{S,13} = $	$= \begin{bmatrix} 0.2849 \\ -0.2477 + 0.0400i \\ 0.0073 - 0.2296i \\ 0.0167 - 0.2654i \end{bmatrix}$	$\begin{array}{c} -0.2477 - 0.0400i\\ 0.2042\\ 0.0104 + 0.2183i\\ -0.0427 + 0.2274i\end{array}$	$\begin{array}{c} 0.0073 + 0.2296i \\ 0.0104 - 0.2183i \\ 0.2687 \\ 0.2297 + 0.0240i \end{array}$	$ \begin{bmatrix} 0.0167 + 0.2654i \\ -0.0427 - 0.2274i \\ 0.2297 - 0.0240i \\ 0.2422 \end{bmatrix} $
$ \rho_{S,14} = $	$\begin{bmatrix} 0.3786\\ -0.0779 + 0.3764i\\ -0.1289 - 0.1049i\\ 0.1243 - 0.0775i \end{bmatrix}$	$\begin{array}{r} -0.0779 - 0.3764i \\ 0.3745 \\ -0.0776 + 0.1422i \\ -0.0950 - 0.1000i \end{array}$	$\begin{array}{c} -0.1289 + 0.1049i \\ -0.0776 - 0.1422i \\ 0.1345 \\ -0.0377 + 0.1163i \end{array}$	$\begin{array}{c} 0.1243 + 0.0775i \\ -0.0950 + 0.1000i \\ -0.0377 - 0.1163i \\ 0.1124 \end{array}$
$ \rho_{S,15} = $	$\begin{bmatrix} 0.5241 \\ 0.1701 - 0.4098i \\ -0.0732 - 0.0497i \\ -0.0613 + 0.0740i \end{bmatrix}$	$\begin{array}{c} 0.1701 + 0.4098i \\ 0.3733 \\ 0.0048 - 0.0800i \\ -0.0776 - 0.0176i \end{array}$	$\begin{array}{c} -0.0732 + 0.0497 i \\ 0.0048 + 0.0800 i \\ 0.0586 \\ 0.0165 - 0.0519 i \end{array}$	$\begin{array}{c} -0.0613 - 0.0740i \\ -0.0776 + 0.0176i \\ 0.0165 + 0.0519i \\ 0.0440 \end{array}$
$ \rho_{S,16} = $	$\begin{bmatrix} 0.0079 \\ -0.0088 + 0.1191i \\ -0.0026 - 0.0469i \\ -0.0179 + 0.0369i \end{bmatrix}$	$\begin{array}{c} -0.0088-0.1191i\\ 0.6443\\ -0.4307+0.0597i\\ -0.0144+0.0826i\end{array}$	$\begin{array}{c} -0.0026+0.0469i\\ -0.4307-0.0597i\\ 0.3254\\ 0.0248-0.0298i\end{array}$	$\begin{array}{c} -0.0179 - 0.0369i \\ -0.0144 - 0.0826i \\ 0.0248 + 0.0298i \\ 0.0224 \end{array}$
$ \rho_{S,17} = $	$\begin{bmatrix} 0.0071 \\ -0.0098 + 0.1224i \\ -0.0079 - 0.0450i \\ -0.0180 + 0.0331i \end{bmatrix}$	$\begin{array}{r} -0.0098 - 0.1224i \\ 0.7482 \\ -0.3769 + 0.0542i \\ -0.0215 + 0.1008i \end{array}$	$\begin{array}{c} -0.0079 + 0.0450i \\ -0.3769 - 0.0542i \\ 0.2232 \\ 0.0274 - 0.0197i \end{array}$	$\begin{array}{c} -0.0180 - 0.0331i \\ -0.0215 - 0.1008i \\ 0.0274 + 0.0197i \\ 0.0215 \end{array}$

$ \rho_{J,1} = $	$\begin{array}{c} 0.4879 \\ -0.0241 - 0.0194i \\ -0.0198 - 0.0473i \\ -0.4503 + 0.0438i \end{array}$	$\begin{array}{c} -0.0241 + 0.0194i \\ 0.0054 \\ -0.0313 - 0.1193i \\ 0.0428 + 0.0066i \end{array}$	$\begin{array}{c} -0.0198 + 0.0473i \\ -0.0313 + 0.1193i \\ 0.0225 \\ -0.0023 - 0.0852i \end{array}$	$ \begin{bmatrix} -0.4503 - 0.0438i \\ 0.0428 - 0.0066i \\ -0.0023 + 0.0852i \\ 0.4842 \end{bmatrix} $
$ \rho_{J,2} =$	$\begin{bmatrix} 0.4748\\ 0.0156 + 0.0702i\\ -0.0009 - 0.0089\\ 0.4543 - 0.0190i \end{bmatrix}$	$\begin{array}{c} 0.0156-0.0702i\\ 0.0107\\i \ 0.0457+0.1052i\\ 0.0248-0.0104i \end{array}$	$\begin{array}{c} -0.0009 + 0.0089i \\ 0.0457 - 0.1052i \\ 0.0079 \\ 0.0098 + 0.0398i \end{array}$	$ \begin{bmatrix} 0.4543 + 0.0190i \\ 0.0248 + 0.0104i \\ 0.0098 - 0.0398i \\ 0.5065 \end{bmatrix} $
$ \rho_{J,3} =$	$\begin{bmatrix} 0.5221 \\ 0.0753 + 0.0707i \\ 0.0706 + 0.0238i \\ 0.0314 - 0.4451i \end{bmatrix}$	$\begin{array}{c} 0.0753-0.0707i\\ 0.0213\\ 0.0016+0.0338i\\ -0.0532-0.0446i \end{array}$	$\begin{array}{c} 0.0706 - 0.0238i \\ 0.0016 - 0.0338i \\ 0.0112 \\ 0.0008 - 0.0618i \end{array}$	$\begin{bmatrix} 0.0314 + 0.4451i \\ -0.0532 + 0.0446i \\ 0.0008 + 0.0618i \\ 0.4454 \end{bmatrix}$
$ \rho_{J,4} = $	$\begin{array}{c} 0.2807\\ 0.2535-0.0302i\\ 0.0392+0.1853i\\ -0.0417-0.2515i\end{array}$	$\begin{array}{c} 0.2535 + 0.0302i \\ 0.2543 \\ -0.0142 + 0.1473i \\ -0.0489 - 0.1999i \end{array}$	$\begin{array}{c} 0.0392 - 0.1853i \\ -0.0142 - 0.1473i \\ 0.1256 \\ -0.2106 + 0.0075i \end{array}$	$\begin{array}{c} -0.0417 + 0.2515i \\ -0.0489 + 0.1999i \\ -0.2106 - 0.0075i \\ 0.3394 \end{array} \right]$
$ \rho_{J,5} = $	$\begin{array}{c} 0.0260 \\ -0.0955 - 0.0352i \\ 0.0370 - 0.0450i \\ 0.0842 - 0.0143i \end{array}$	$\begin{array}{c} -0.0955 + 0.0352i\\ 0.5035\\ 0.0139 + 0.4182i\\ -0.0177 + 0.0134i\end{array}$	$\begin{array}{c} 0.0370 + 0.0450 i \\ 0.0139 - 0.4182 i \\ 0.4676 \\ -0.0199 + 0.0215 i \end{array}$	$ \begin{bmatrix} 0.0842 + 0.0143i \\ -0.0177 - 0.0134i \\ -0.0199 - 0.0215i \\ 0.0029 \end{bmatrix} $
$\rho_{J,6} =$	$\begin{bmatrix} 0.0367 \\ 0.0389 - 0.1059i \\ 0.0675 - 0.0395i \\ 0.0312 - 0.0146i \end{bmatrix}$	$\begin{array}{c} 0.0389 + 0.1059i \\ 0.4500 \\ 0.4370 + 0.0716i \\ -0.0533 + 0.0099i \end{array}$	$\begin{array}{c} 0.0675 + 0.0395i\\ 0.4370 - 0.0716i\\ 0.5064\\ -0.0482 + 0.0141i\end{array}$	$ \begin{bmatrix} 0.0312 + 0.0146i \\ -0.0533 - 0.0099i \\ -0.0482 - 0.0141i \\ 0.0069 \end{bmatrix} $
$\rho_{J,7} =$	$\begin{bmatrix} 0.0062 \\ 0.0456 + 0.0314i \\ 0.0057 - 0.0030i \\ 0.0072 + 0.0079i \end{bmatrix}$	$\begin{array}{c} 0.0456 - 0.0314i \\ 0.9818 \\ 0.0226 - 0.0910i \\ -0.0969 + 0.0714i \end{array}$	$\begin{array}{c} 0.0057 + 0.0030i\\ 0.0226 + 0.0910i\\ 0.0032\\ -0.0035 - 0.0032i\end{array}$	$ \begin{bmatrix} 0.0072 - 0.0079i \\ -0.0969 - 0.0714i \\ -0.0035 + 0.0032i \\ 0.0087 \end{bmatrix} $
$ \rho_{J,8} = $	$\begin{array}{c} 0.5550 \\ -0.0784 - 0.0200i \\ -0.0058 + 0.0372i \\ -0.0223 + 0.4409i \end{array}$	$\begin{array}{c} -0.0784 + 0.0200i\\ 0.0135\\ 0.0219 - 0.0377i\\ -0.0032 - 0.0374i\end{array}$	$\begin{array}{c} -0.0058 - 0.0372i\\ 0.0219 + 0.0377i\\ 0.0043\\ -0.0027 + 0.0162i\end{array}$	$\begin{array}{c} -0.0223 - 0.4409i \\ -0.0032 + 0.0374i \\ -0.0027 - 0.0162i \\ 0.4271 \end{array} \right]$
$ \rho_{J,9} = $	$\begin{array}{c} 0.0089 \\ -0.0324 + 0.0301i \\ -0.0031 + 0.0189i \\ 0.0009 + 0.0043i \end{array}$	$\begin{array}{c} -0.0324 - 0.0301i\\ 0.5684\\ -0.4348 - 0.0409i\\ -0.0166 + 0.0446i\end{array}$	$\begin{array}{c} -0.0031 - 0.0189i \\ -0.4348 + 0.0409i \\ 0.4209 \\ 0.0002 + 0.0151i \end{array}$	$ \begin{bmatrix} 0.0009 - 0.0043i \\ -0.0166 - 0.0446i \\ 0.0002 - 0.0151i \\ 0.0018 \end{bmatrix} $
$ \rho_{J,10} = $	$\begin{bmatrix} 0.0058\\ 0.0400 + 0.0019i\\ 0.0128 + 0.0240i\\ -0.0401 + 0.0454i \end{bmatrix}$	0.0400 - 0.0019i 0.5133 0.0953 - 0.3781i i - 0.0293 - 0.0144	$\begin{array}{c} 0.0128-0.0240i\\ 0.0953+0.3781i\\ 0.4753\\ i \ 0.0619-0.0094i \end{array}$	$\begin{array}{c} -0.0401 - 0.0454i \\ -0.0293 + 0.0144i \\ 0.0619 + 0.0094i \\ 0.0056 \end{array} \right]$

4

$$\rho_{J,11} = \begin{bmatrix} 0.2918 & 0.2453 - 0.0200i & -0.0285 + 0.1767i & -0.0527 - 0.2169i \\ 0.2453 + 0.0200i & 0.1992 & 0.0202 + 0.2358i & 0.0220 - 0.2014i \\ -0.0285 - 0.1767i & 0.0202 - 0.2358i & 0.2948 & -0.2464 + 0.0008i \\ -0.0527 + 0.2169i & 0.0220 + 0.2014i & -0.2464 - 0.0008i & 0.2142 \end{bmatrix}$$

$$\rho_{J,12} = \begin{bmatrix} 0.1923 & -0.2367 - 0.0390i & -0.0338 - 0.1873i & -0.0453 - 0.2798i^{-1} \\ -0.2367 + 0.0390i & 0.3002 & 0.0996 + 0.1912i & 0.0039 + 0.2219i \\ -0.0338 + 0.1873i & 0.0996 - 0.1912i & 0.2127 & 0.2404 + 0.0740i \\ -0.0453 + 0.2798i & 0.0039 - 0.2219i & 0.2404 - 0.0740i & 0.2947 \end{bmatrix}$$

$$\rho_{J,13} = \begin{bmatrix} 0.3064 & -0.2350 - 0.0157i & 0.0162 + 0.1740i & 0.0769 + 0.2162i \\ -0.2350 + 0.0157i & 0.1884 & -0.0753 - 0.2582i & -0.0327 - 0.1833i \\ 0.0162 - 0.1740i & -0.0753 + 0.2582i & 0.2840 & 0.2538 - 0.0131i \\ 0.0769 - 0.2162i & -0.0327 + 0.1833i & 0.2538 + 0.0131i & 0.2211 \end{bmatrix}$$

$$\rho_{J,14} = \begin{bmatrix} 0.3803 & -0.0676 - 0.4027i & -0.1333 + 0.1142i & 0.1197 + 0.1053i \\ -0.0676 + 0.4027i & 0.3748 & -0.0745 - 0.1464i & -0.0932 + 0.1079i \\ -0.1333 - 0.1142i & -0.0745 + 0.1464i & 0.1345 & -0.0335 - 0.1211i \\ 0.1197 - 0.1053i & -0.0932 - 0.1079i & -0.0335 + 0.1211i & 0.1104 \end{bmatrix}$$

$$\rho_{J,15} = \begin{bmatrix} 0.5276 & 0.1690 + 0.4181i & -0.0627 + 0.0920i & -0.0903 - 0.0653i \\ 0.1690 - 0.4181i & 0.3819 & 0.0342 + 0.0807i & -0.0764 + 0.0379i \\ -0.0627 - 0.0920i & 0.0342 - 0.0807i & 0.0478 & 0.0032 + 0.0440i \\ -0.0903 + 0.0653i & -0.0764 - 0.0379i & 0.0032 - 0.0440i & 0.0427 \end{bmatrix}$$

$$\rho_{J,16} = \begin{bmatrix} 0.0207 & -0.0045 - 0.1146i & -0.0112 + 0.0073i & -0.0325 - 0.0020i \\ -0.0045 + 0.1146i & 0.6312 & -0.4297 + 0.0089i & 0.0034 - 0.0464i \\ -0.0112 - 0.0073i & -0.4297 - 0.0089i & 0.3450 & 0.0040 + 0.0183i \\ -0.0325 + 0.0020i & 0.0034 + 0.0464i & 0.0040 - 0.0183i & 0.0031 \end{bmatrix}$$

$$\rho_{J,17} = \begin{bmatrix} 0.0202 & -0.0172 - 0.1220i & -0.0095 + 0.0268i & -0.0629 - 0.0175i \\ -0.0172 + 0.1220i & 0.7415 & -0.3322 - 0.0029i & -0.0138 - 0.0503i \\ -0.0095 - 0.0268i & -0.3322 + 0.0029i & 0.2341 & 0.0108 - 0.0009i \\ -0.0629 + 0.0175i & -0.0138 + 0.0503i & 0.0108 + 0.0009i & 0.0042 \end{bmatrix}$$

C. MUB-based tomography

	Г 0.4789	0.0857 + 0.0275i	0.0044 + 0.0534i	-0.4699 - 0.0476i	
~ —	0.0857 - 0.0275i	0.0311	-0.0110 - 0.0029i	0.0332 - 0.0196i	
$\rho_{M,1} =$	0.0044 - 0.0534i	-0.0110 + 0.0029i	-0.0015	0.0215 + 0.0758i	
	-0.4699 + 0.0476i	0.0332 + 0.0196i	0.0215 - 0.0758i	0.4915	
	Г 0.4888	-0.0083 - 0.0456i	0.0198 + 0.0013i	0.4919 + 0.0541i	
		0.0000	0.0007 0.0011:	0.0000 + 0.0001 *	I.

$$\rho_{M,2} = \begin{bmatrix} 0.4888 & -0.0083 - 0.0436i & 0.0198 + 0.0013i & 0.4919 + 0.0541i \\ -0.0083 + 0.0456i & 0.0296 & 0.0027 - 0.0241i & 0.0206 + 0.0291i \\ 0.0198 - 0.0013i & 0.0027 + 0.0241i & -0.0322 & -0.0820 - 0.0477i \\ 0.4919 - 0.0541i & 0.0206 - 0.0291i & -0.0820 + 0.0477i & 0.5138 \end{bmatrix}$$

$$\begin{split} \rho_{M,3} &= \begin{bmatrix} 0.5214 & 0.0880 - 0.0452i & 0.0855 - 0.0535i & 0.05614 + 0.4566i \\ 0.0865 + 0.0452i & 0.0073 & -0.0016i & -0.0472 + 0.0870i \\ 0.0865 + 0.0535i & -0.0021 - 0.0016i & 0.0053 & -0.0563 + 0.0557i \\ 0.0504 - 0.0472 - 0.0870i & -0.0663 - 0.0557i & 0.4160 \end{bmatrix} \\ \rho_{M,4} &= \begin{bmatrix} 0.2423 & 0.2560 + 0.0449i & 0.0503 - 0.2014i & 0.0328 + 0.2533i \\ 0.2560 - 0.0449i & 0.3304 & 0.1157 - 0.1609i & -0.0336 + 0.2319i \\ 0.0503 - 0.2014i & 0.1157 + 0.1609i & 0.1676 & -0.1754 + 0.0308i \\ 0.0328 - 0.2533i & -0.0366 - 0.2319i & -0.1754 + 0.0308i & 0.2597 \end{bmatrix} \\ \rho_{M,5} &= \begin{bmatrix} -0.0004 & -0.0941 + 0.0188i & 0.0384 + 0.0899i & 0.0017 - 0.0313i \\ 0.0384 - 0.0399i & 0.1830 + 0.4264i & 0.5160 & 0.0302 - 0.0038i \\ 0.0017 + 0.0313i & 0.0052 + 0.0638i & 0.0302 + 0.0058i & -0.0131 \end{bmatrix} \\ \rho_{M,6} &= \begin{bmatrix} 0.0119 & 0.0116 + 0.0964i & 0.0767 + 0.0744i & 0.0106 + 0.0150i \\ 0.016 - 0.0964i & 0.4477 & 0.4659 + 0.0031i & -0.0301 - 0.0330i \\ 0.0767 - 0.0744i & 0.4659 - 0.0031i & 0.0552 & -0.0780 - 0.0024i \\ 0.0106 - 0.0150i & -0.0301 + 0.0330i & -0.0780 + 0.0024i & -0.0121 \end{bmatrix} \\ \rho_{M,7} &= \begin{bmatrix} 0.0377 & 0.0294 - 0.0443i & 0.0928 & 0.0006 + 0.0373i & 0.0462 - 0.1062i \\ 0.0058 + 0.0081i & 0.0066 - 0.0373i & 0.0075 + 0.0315i \\ 0.0058 + 0.0051i & -0.0301i & 0.0324 + 0.0106i & -0.0598 & -0.494i \\ 0.0058 - 0.0214i & 0.0052 + 0.0106ii & 0.0296 - 0.0029i & -0.0052 \end{bmatrix} \end{bmatrix} \\ \rho_{M,8} &= \begin{bmatrix} 0.5549 & -0.0058 + 0.0214i & 0.0324 + 0.0106i & -0.0598 - 0.4494i \\ 0.0035 - 0.0214i & 0.0055 + 0.0171i & -0.0329 & -0.0199 + 0.0231i \\ -0.0058 + 0.0434i & 0.0052 + 0.0174i & -0.0328i & 0.0075 + 0.0315i \\ -0.0028 - 0.0180i & -0.0676 + 0.0233i & 0.0472 + 0.0406i \\ -0.0028 - 0.0180i & -0.0676 + 0.0238i & 0.0788 + 0.047i & 0.0228 \end{bmatrix} \end{bmatrix} \\ \rho_{M,10} &= \begin{bmatrix} -0.0020 + 0.0764i & 0.5405 & -0.074i & -0.0129i & 0.0321i \\ -0.0029 + 0.0433i & 0.0676 + 0.0233i & 0.0798 + 0.047i & 0.0228 \end{bmatrix} \end{bmatrix} \\ \rho_{M,11} &= \begin{bmatrix} 0.2210 & -0.0415 - 0.0236i & 0.0226i & 0.2387i & 0.0049 - 0.2545i \\ -0.0228 - 0.0180i & -0.0266 + 0.2206i & 0.2285 & -0.0228i \\ -0.0228 - 0.0238i & 0.0076 + 0.2359i & 0.0798 + 0.0047i & 0.2218i \\ 0.002$$

$$\rho_{M,13} = \begin{bmatrix} 0.3059 & -0.2612 - 0.0147i & 0.0146 + 0.2246i & 0.0422 + 0.2544i \\ -0.2612 + 0.0147i & 0.1762 & -0.0175 - 0.2085i & -0.0342 - 0.2370i \\ 0.0146 - 0.2246i & -0.0175 + 0.2085i & 0.2792 & 0.1911 + 0.0008i \\ 0.0422 - 0.2544i & -0.0342 + 0.2370i & 0.1911 - 0.0008i & 0.2387 \end{bmatrix}$$

$$\rho_{M,14} = \begin{bmatrix} 0.4095 & -0.0753 - 0.3984i & -0.1351 + 0.1251i & 0.1159 + 0.0932i \\ -0.0753 + 0.3984i & 0.3649 & -0.0998 - 0.1619i & -0.0991 + 0.1004i \\ -0.1351 - 0.1251i & -0.0998 + 0.1619i & 0.1005 & -0.0384 - 0.1226i \\ 0.1159 - 0.0932i & -0.0991 - 0.1004i & -0.0384 + 0.1226i & 0.1251 \end{bmatrix}$$

$$\rho_{M,15} = \begin{bmatrix} 0.5590 & 0.1688 + 0.4115i & -0.0750 + 0.0461i & -0.0342 - 0.0741i \\ 0.1688 - 0.4115i & 0.3472 & 0.0472 + 0.0803i & -0.0795 + 0.0198i \\ -0.0750 - 0.0461i & 0.0472 - 0.0803i & 0.0258 & 0.0397 + 0.0409i \\ -0.0342 + 0.0741i & -0.0795 - 0.0198i & 0.0397 - 0.0409i & 0.0680 \end{bmatrix}$$

$$\rho_{M,16} = \begin{bmatrix} 0.0095 & -0.0057 - 0.1331i & -0.0030 + 0.0330i & -0.0239 - 0.0074i \\ -0.0057 + 0.1331i & 0.6278 & -0.3716 - 0.0931i & -0.0152 - 0.0669i \\ -0.0030 - 0.0330i & -0.3716 + 0.0931i & 0.3255 & 0.0919 + 0.0219i \\ -0.0239 + 0.0074i & -0.0152 + 0.0669i & 0.0919 - 0.0219i & 0.0372 \end{bmatrix}$$

$$\rho_{M,17} = \begin{bmatrix} 0.0219 & -0.0122 - 0.1428i & -0.0034 + 0.0132i & -0.0230 - 0.0034i \\ -0.0122 + 0.1428i & 0.7250 & -0.3510 - 0.0873i & -0.0176 - 0.0751i \\ -0.0034 - 0.0132i & -0.3510 + 0.0873i & 0.2151 & 0.0860 + 0.0050i \\ -0.0230 + 0.0034i & -0.0176 + 0.0751i & 0.0860 - 0.0050i & 0.0379 \end{bmatrix}$$

D. Optimal tomography

	Г 0.4879	-0.0194 + 0.0278i	0.0045 + 0.0413i	-0.4760 - 0.0086i
$ \rho_{O,1} =$	-0.0194 - 0.0278i	0.0054	-0.0112 - 0.0003i	0.0336 + 0.0064i
	0.0045 - 0.0413i	-0.0112 + 0.0003i	0.0225	-0.0033 + 0.0768i
	$\lfloor -0.4760 + 0.0086i \rfloor$	0.0336 - 0.0064i	-0.0033 - 0.0768i	0.4842

	Г 0.4748	0.0191 - 0.0443i	0.0193 - 0.0130i	0.4781 + 0.0019i
$ \rho_{O,2} =$	0.0191 + 0.0443i	0.0107	0.0026 + 0.0001i	0.0200 + 0.0379i
	0.0193 + 0.0130i	0.0026 - 0.0001i	0.0079	0.0111 - 0.0464i
	0.4781 - 0.0019i	0.0200 - 0.0379i	0.0111 + 0.0464i	0.5065

	Г 0.5221	0.0816 - 0.0442i	0.0847 - 0.0240i	0.0530 + 0.4532i]
$ \rho_{O,3} = $	0.0816 + 0.0442i	0.0213	-0.0020 - 0.0026i	-0.0463 + 0.0584i
	0.0847 + 0.0240i	-0.0020 + 0.0026i	0.0112	0.0095 + 0.0545i
	0.0530 - 0.4532i	-0.0463 - 0.0584i	0.0095 - 0.0545i	0.4454

	Г 0.2807	0.2569 + 0.0406i	0.0456 - 0.2118i	0.0297 + 0.2155i
$\rho_{O,4} =$	0.2569 - 0.0406i	0.2543	0.1047 - 0.1856i	-0.0304 + 0.2331i
	0.0456 + 0.2118i	0.1047 + 0.1856i	0.1256	-0.2027 - 0.0279i
	0.0297 - 0.2155i	-0.0304 - 0.2331i	-0.2027 + 0.0279i	0.3394

$ \rho_{O,5} = $	$\begin{bmatrix} 0.0260 \\ -0.0969 - 0.0186i \\ 0.0380 - 0.0730i \\ 0.0017 + 0.0078i \end{bmatrix}$	$\begin{array}{c} -0.0969 + 0.0186i \\ 0.5035 \\ 0.1813 + 0.4222i \\ 0.0051 + 0.0374i \end{array}$	$\begin{array}{c} 0.0380 + 0.0730i\\ 0.1813 - 0.4222i\\ 0.4676\\ -0.0051 + 0.0057i \end{array}$	$ \begin{bmatrix} 0.0017 - 0.0078i \\ 0.0051 - 0.0374i \\ -0.0051 - 0.0057i \\ 0.0029 \end{bmatrix} $
$ \rho_{O,6} = $	$\begin{bmatrix} 0.0367 \\ 0.0502 - 0.0923i \\ 0.0734 - 0.0820i \\ 0.0102 + 0.0178i \end{bmatrix}$	$\begin{array}{c} 0.0502 + 0.0923i \\ 0.4500 \\ 0.4460 + 0.0748i \\ -0.0288 + 0.0230i \end{array}$	$\begin{array}{c} 0.0734 + 0.0820i \\ 0.4460 - 0.0748i \\ 0.5064 \\ -0.0430 + 0.0023i \end{array}$	$ \begin{bmatrix} 0.0102 - 0.0178i \\ -0.0288 - 0.0230i \\ -0.0430 - 0.0023i \\ 0.0069 \end{bmatrix} $
$ \rho_{O,7} = $	$\begin{bmatrix} 0.0062 \\ 0.0557 + 0.0422i \\ 0.0047 - 0.0036i \\ -0.0072 - 0.0029i \end{bmatrix}$	$\begin{array}{c} 0.0557-0.0422i\\ 0.9818\\ 0.0006-0.0380i\\ -0.0441+0.1031i \end{array}$	$\begin{array}{c} 0.0047 + 0.0036i \\ 0.0006 + 0.0380i \\ 0.0032 \\ -0.0049 - 0.0027i \end{array}$	$\begin{array}{c} -0.0072 + 0.0029i \\ -0.0441 - 0.1031i \\ -0.0049 + 0.0027i \\ 0.0087 \end{array} \right]$
$ \rho_{O,8} = $	$\begin{bmatrix} 0.5550 \\ -0.0622 - 0.0204i \\ 0.0308 + 0.0306i \\ -0.0569 + 0.4699i \end{bmatrix}$	$\begin{array}{c} -0.0622 + 0.0204i \\ 0.0135 \\ 0.0053 + 0.0015i \\ -0.0050 - 0.0533i \end{array}$	$\begin{array}{c} 0.0308-0.0306i\\ 0.0053-0.0015i\\ 0.0043\\ -0.0039+0.0220i\end{array}$	$\begin{array}{c} -0.0569 - 0.4699i \\ -0.0050 + 0.0533i \\ -0.0039 - 0.0220i \\ 0.4271 \end{array} \right]$
$ \rho_{O,9} =$	$\begin{bmatrix} 0.0089 \\ -0.0219 + 0.0731i \\ -0.0027 - 0.0214i \\ -0.0130 - 0.0040i \end{bmatrix}$	$\begin{array}{c} -0.0219 - 0.0731i \\ 0.5684 \\ -0.4492 - 0.0011i \\ -0.0065 + 0.0551i \end{array}$	$\begin{array}{c} -0.0027 + 0.0214i \\ -0.4492 + 0.0011i \\ 0.4209 \\ -0.0068 + 0.0044i \end{array}$	$\begin{array}{c} -0.0130 + 0.0040i \\ -0.0065 - 0.0551i \\ -0.0068 - 0.0044i \\ 0.0018 \end{array} \right]$
$ \rho_{O,10} = $	$\begin{bmatrix} 0.0058\\ 0.0406 + 0.0232i\\ 0.0029 - 0.0025i\\ -0.0009 - 0.0092i \end{bmatrix}$	$\begin{array}{c} 0.0406 - 0.0232i\\ 0.5133\\ -0.0158 - 0.4665i\\ -0.0279 + 0.0088i\end{array}$	$\begin{array}{c} 0.0029 + 0.0025 i \\ -0.0158 + 0.4665 i \\ 0.4753 \\ 0.0465 - 0.0408 i \end{array}$	$ \begin{array}{c} -0.0009 + 0.0092i \\ -0.0279 - 0.0088i \\ 0.0465 + 0.0408i \\ 0.0056 \end{array} \right] $
$ \rho_{O,11} = $	$\begin{bmatrix} 0.2918\\ 0.2395 + 0.0021i\\ -0.0242 - 0.2258i\\ 0.0050 + 0.2663i \end{bmatrix}$	$\begin{array}{c} 0.2395 - 0.0021i\\ 0.1992\\ -0.0274 - 0.2259i\\ 0.0132 + 0.2382i \end{array}$	$\begin{array}{c} -0.0242 + 0.2258i \\ -0.0274 + 0.2259i \\ 0.2948 \\ -0.2469 + 0.0169i \end{array}$	$ \begin{bmatrix} 0.0050 - 0.2663i \\ 0.0132 - 0.2382i \\ -0.2469 - 0.0169i \\ 0.2142 \end{bmatrix} $
$ \rho_{O,12} = $	$\begin{bmatrix} 0.1923 \\ -0.2541 + 0.0502i \\ -0.0467 + 0.2470i \\ -0.0163 + 0.2401i \end{bmatrix}$	$\begin{array}{c} -0.2541 - 0.0502i \\ 0.3002 \\ 0.0111 - 0.2257i \\ 0.0075 - 0.2518i \end{array}$	$\begin{array}{c} -0.0467 - 0.2470 i \\ 0.0111 + 0.2257 i \\ 0.2127 \\ 0.2396 - 0.0827 i \end{array}$	$ \begin{bmatrix} -0.0163 - 0.2401i \\ 0.0075 + 0.2518i \\ 0.2396 + 0.0827i \\ 0.2947 \end{bmatrix} $
$ \rho_{O,13} = $	$\begin{bmatrix} 0.3064 \\ -0.2359 + 0.0153i \\ 0.0152 - 0.2270i \\ 0.0439 - 0.2918i \end{bmatrix}$	$\begin{array}{c} -0.2359 - 0.0153i\\ 0.1884\\ -0.0182 + 0.1992i\\ -0.0356 + 0.2367i\end{array}$	$\begin{array}{c} 0.0152 + 0.2270i \\ -0.0182 - 0.1992i \\ 0.2840 \\ 0.2487 - 0.0009i \end{array}$	$ \begin{bmatrix} 0.0439 + 0.2918i \\ -0.0356 - 0.2367i \\ 0.2487 + 0.0009i \\ 0.2211 \end{bmatrix} $
$ \rho_{O,14} = $	$\begin{bmatrix} 0.3803 \\ -0.0814 + 0.3887i \\ -0.1318 - 0.1051i \\ 0.1131 - 0.0665i \end{bmatrix}$	$\begin{array}{c} -0.0814 - 0.3887i \\ 0.3748 \\ -0.0973 + 0.1265i \\ -0.0967 - 0.1000i \end{array}$	$\begin{array}{c} -0.1318 + 0.1051 i \\ -0.0973 - 0.1265 i \\ 0.1345 \\ -0.0398 + 0.1196 i \end{array}$	$\begin{array}{c} 0.1131 + 0.0665i \\ -0.0967 + 0.1000i \\ -0.0398 - 0.1196i \\ 0.1104 \end{array} \right]$

$$\rho_{O,15} = \begin{bmatrix} 0.5276 & 0.1680 + 0.4105i & -0.0748 + 0.0527i & -0.0341 - 0.0538i \\ 0.1680 - 0.4105i & 0.3819 & 0.0471 + 0.0874i & -0.0793 + 0.0196i \\ -0.0748 - 0.0527i & 0.0471 - 0.0874i & 0.0478 & 0.0094 + 0.0408i \\ -0.0341 + 0.0538i & -0.0793 - 0.0196i & 0.0094 - 0.0408i & 0.0427 \end{bmatrix}$$

$$\rho_{O,16} = \begin{bmatrix} 0.0207 & -0.0261 - 0.1296i & -0.0029 + 0.0401i & -0.0233 - 0.0012i \\ -0.0261 + 0.1296i & 0.6312 & -0.3619 - 0.0096i & -0.0148 - 0.0912i \\ -0.0029 - 0.0401i & -0.3619 + 0.0096i & 0.3450 & 0.0080 + 0.0213i \\ -0.0233 + 0.0012i & -0.0148 + 0.0912i & 0.0080 - 0.0213i & 0.0031 \end{bmatrix}$$

$$\rho_{O,17} = \begin{bmatrix} 0.0202 & -0.0269 - 0.1385i & -0.0033 + 0.0486i & -0.0223 + 0.0155i \\ -0.0269 + 0.1385i & 0.7415 & -0.3405 + 0.0102i & -0.0171 - 0.0985i \\ -0.0033 - 0.0486i & -0.3405 - 0.0102i & 0.2341 & 0.0106 + 0.0049i \end{bmatrix}$$

E. Pauli matrices based tomography

-0.0223 - 0.0155i -0.0171 + 0.0985i 0.0106 - 0.0049i

 $\begin{bmatrix} 0.4879 & -0.0194 + 0.0278i & 0.0045 + 0.0413i & -0.4516 - 0.0735i \\ -0.0194 - 0.0278i & 0.0054 & -0.0053 + 0.0223i & 0.0336 + 0.0064i \\ 0.0045 - 0.0413i & -0.0053 - 0.0223i & 0.0225 & -0.0033 + 0.0768i \\ \end{bmatrix}$ -0.0194 + 0.0278i 0.0045 + 0.0413i -0.4516 - 0.0735i-0.0033 + 0.0768i-0.4516 + 0.0735i 0.0336 - 0.0064i -0.0033 - 0.0768i0.48420.0193 - 0.0130i 0.4662 + 0.0508i0.0191 - 0.0443i0.4748 $\begin{vmatrix} 0.0191 + 0.0443i & 0.0107 & -0.0000 - 0.0216i & 0.0200 + 0.0379i \\ 0.0193 + 0.0130i & -0.0000 + 0.0216i & 0.0079 & 0.0111 - 0.0464i \end{vmatrix}$ $0.4662 - 0.0508i \quad 0.0200 - 0.0379i$ 0.0111 + 0.0464i0.5065 $\begin{bmatrix} 0.5221 & 0.0816 - 0.0442i \\ 0.0816 + 0.0442i & 0.0213 \\ 0.0847 + 0.0240i & 0.0509 + 0.0035i \end{bmatrix}$ 0.0847 - 0.0240i - 0.0026 + 0.4523i0.0509 - 0.0035i - 0.0463 + 0.0584i0.0095 + 0.0545i0.0112-0.0026 - 0.4523i - 0.0463 - 0.0584i 0.0095 - 0.0545i0.4454 $\begin{bmatrix} 0.2807 & 0.2569 + 0.0406i \\ 0.2569 - 0.0406i & 0.2543 \\ 0.0456 + 0.2118i & 0.0401 + 0.1770i \end{bmatrix}$ 0.0456 - 0.2118i - 0.0623 + 0.2607i0.0401 - 0.1770i - 0.0304 + 0.2331i0.1256-0.2027 - 0.0279i-0.0623 - 0.2607i -0.0304 - 0.2331i -0.2027 + 0.0279i0.3394-0.0969 + 0.0186i0.0380 + 0.0730i0.0156 + 0.0018i0.0260 $\rho_{P,5} = \begin{vmatrix} -0.0969 - 0.0186i & 0.5035 \\ 0.0380 - 0.0730i & 0.0389 + 0.4553i \end{vmatrix}$ 0.0051 - 0.0374i0.50350.0389 - 0.4553i0.4676-0.0051 - 0.0057i0.0156 - 0.0018i0.0051 + 0.0374i-0.0051 + 0.0057i0.00290.03670.0502 + 0.0923i0.0734 + 0.0820i-0.0044 + 0.0326i0.0502 - 0.0923i0.45000.4397 - 0.0152i-0.0288 - 0.0230i $\rho_{P,6} =$ 0.0734 - 0.0820i0.4397 + 0.0152i0.5064-0.0430 - 0.0023i-0.0044 - 0.0326i -0.0288 + 0.0230i -0.0430 + 0.0023i0.0069 0.0557 - 0.0422i0.0047 + 0.0036i - 0.0086 - 0.0005i0.0062-0.0441 - 0.1031i-0.0049 + 0.0027i0.0087

0.0042

$\rho_{P,8} =$	$\begin{bmatrix} 0.5550 \\ -0.0622 - 0.0204i \\ 0.0308 + 0.0306i \\ 0.0026 + 0.4514i \end{bmatrix}$	$\begin{array}{r} -0.0622 + 0.0204i \\ 0.0135 \\ -0.0451 - 0.0076i \\ -0.0050 - 0.0533i \end{array}$	$\begin{array}{c} 0.0308 - 0.0306i \\ -0.0451 + 0.0076i \\ 0.0043 \\ -0.0039 + 0.0220i \end{array}$	$ \begin{bmatrix} 0.0026 - 0.4514i \\ -0.0050 + 0.0533i \\ -0.0039 - 0.0220i \\ 0.4271 \end{bmatrix} $
$ \rho_{P,9} =$	$\begin{bmatrix} 0.0089 \\ -0.0219 + 0.0731i \\ -0.0027 - 0.0214i \\ -0.0215 + 0.0387i \end{bmatrix}$	$\begin{array}{c} -0.0219 - 0.0731i \\ 0.5684 \\ -0.4398 + 0.0689i \\ -0.0065 + 0.0551i \end{array}$	$\begin{array}{c} -0.0027 + 0.0214i \\ -0.4398 - 0.0689i \\ 0.4209 \\ -0.0068 + 0.0044i \end{array}$	$\begin{array}{c} -0.0215-0.0387i\\ -0.0065-0.0551i\\ -0.0068-0.0044i\\ 0.0018 \end{array} \right]$
$\rho_{P,10} =$	$= \begin{bmatrix} 0.0058\\ 0.0406 + 0.0232i\\ 0.0029 - 0.0025i\\ -0.0090 + 0.0064i \end{bmatrix}$	$\begin{array}{c} 0.0406-0.0232i\\ 0.5133\\ 0.0500-0.4689i\\ i\\ -0.0279+0.0088i\end{array}$	$\begin{array}{c} 0.0029 + 0.0025 i \\ 0.0500 + 0.4689 i \\ 0.4753 \\ 0.0465 - 0.0408 i \end{array}$	$ \begin{bmatrix} -0.0090 - 0.0064i \\ -0.0279 - 0.0088i \\ 0.0465 + 0.0408i \\ 0.0056 \end{bmatrix} $
$ \rho_{P,11} = $	$\begin{bmatrix} 0.2918 \\ 0.2395 + 0.0021i \\ -0.0242 - 0.2258i \\ 0.0052 + 0.2556i \end{bmatrix}$	$\begin{array}{c} 0.2395 - 0.0021i \\ 0.1992 \\ -0.0291 - 0.2209i \\ 0.0132 + 0.2382i \end{array}$	$\begin{array}{c} -0.0242 + 0.2258i \\ -0.0291 + 0.2209i \\ 0.2948 \\ -0.2469 + 0.0169i \end{array}$	$\begin{array}{c} 0.0052 - 0.2556i \\ 0.0132 - 0.2382i \\ -0.2469 - 0.0169i \\ 0.2142 \end{array}$
$\rho_{P,12} =$	$= \begin{bmatrix} 0.1923 \\ -0.2541 + 0.0502i \\ -0.0467 + 0.2470i \\ 0.0350 + 0.2186i \end{bmatrix}$	$\begin{array}{c} -0.2541 - 0.0502i\\ 0.3002\\ \hline 0.0458 - 0.2493i\\ 0.0075 - 0.2518i \end{array}$	$\begin{array}{c} -0.0467 - 0.2470i \\ 0.0458 + 0.2493i \\ 0.2127 \\ 0.2396 - 0.0827i \end{array}$	$\begin{bmatrix} 0.0350 - 0.2186i \\ 0.0075 + 0.2518i \\ 0.2396 + 0.0827i \\ 0.2947 \end{bmatrix}$
$\rho_{P,13} =$	$= \begin{bmatrix} 0.3064 \\ -0.2359 + 0.0153i \\ 0.0152 - 0.2270i \\ 0.0170 - 0.2694i \end{bmatrix}$	$\begin{array}{c} -0.2359 - 0.0153i\\ 0.1884\\ 0.0105 + 0.2216i\\ -0.0356 + 0.2367i\end{array}$	$\begin{array}{c} 0.0152 + 0.2270i \\ 0.0105 - 0.2216i \\ 0.2840 \\ 0.2487 - 0.0009i \end{array}$	$ \begin{bmatrix} 0.0170 + 0.2694i \\ -0.0356 - 0.2367i \\ 0.2487 + 0.0009i \\ 0.2211 \end{bmatrix} $
$ \rho_{P,14} = $	$\begin{bmatrix} 0.3803 \\ -0.0814 + 0.3887i \\ -0.1318 - 0.1051i \\ 0.1286 - 0.0801i \end{bmatrix}$	$\begin{array}{r} -0.0814 - 0.3887i \\ 0.3748 \\ -0.0803 + 0.1471i \\ -0.0967 - 0.1000i \end{array}$	$\begin{array}{c} -0.1318 + 0.1051 i \\ -0.0803 - 0.1471 i \\ 0.1345 \\ -0.0398 + 0.1196 i \end{array}$	$\begin{array}{c} 0.1286 + 0.0801i \\ -0.0967 + 0.1000i \\ -0.0398 - 0.1196i \\ 0.1104 \end{array}$
$ \rho_{P,15} = $	$\begin{bmatrix} 0.5276 \\ 0.1680 - 0.4105i \\ -0.0748 - 0.0527i \\ -0.0633 + 0.0764i \end{bmatrix}$	$\begin{array}{c} 0.1680 + 0.4105i \\ 0.3819 \\ 0.0050 - 0.0826i \\ -0.0793 - 0.0196i \end{array}$	$\begin{array}{c} -0.0748 + 0.0527 i \\ 0.0050 + 0.0826 i \\ 0.0478 \\ 0.0094 - 0.0408 i \end{array}$	$\begin{array}{c} -0.0633 - 0.0764i \\ -0.0793 + 0.0196i \\ 0.0094 + 0.0408i \\ 0.0427 \end{array}$
$ \rho_{P,16} = $	$\begin{bmatrix} 0.0207 \\ -0.0261 + 0.1296i \\ -0.0029 - 0.0401i \\ -0.0181 + 0.0374i \end{bmatrix}$	$\begin{array}{c} -0.0261 - 0.1296i \\ 0.6312 \\ -0.4367 + 0.0605i \\ -0.0148 + 0.0912i \end{array}$	$\begin{array}{c} -0.0029 + 0.0401 i \\ -0.4367 - 0.0605 i \\ 0.3450 \\ 0.0080 - 0.0213 i \end{array}$	$\begin{array}{c} -0.0181 - 0.0374i^{-0.0148} \\ -0.0148 - 0.0912i \\ 0.0080 + 0.0213i \\ 0.0031 \end{array}$
$ \rho_{P,17} = $	$\begin{bmatrix} 0.0202 \\ -0.0269 + 0.1385i \\ -0.0033 - 0.0486i \\ -0.0181 + 0.0334i \end{bmatrix}$	-0.0269 - 0.1385i 0.7415 -0.3803 + 0.0547i -0.0171 + 0.0985i	-0.0033 + 0.0486i -0.3803 - 0.0547i 0.2341 0.0106 - 0.0049i	$\begin{array}{c} -0.0181 - 0.0334i^{-} \\ -0.0171 - 0.0985i \\ 0.0106 + 0.0049i \\ 0.0042 \end{array}$

II. COEFFICIENT MATRICES

All the analyzed tomographies are based on solving the linear-system problem

$$Ax = b,$$

where A is the *coefficient matrix*, b is the observation vector, and $x = vec(\rho)$ is a real vector describing the unknown state ρ , i.e.,

$$x = \operatorname{vec}(\rho) = [\rho_{11}, \operatorname{Re}\rho_{12}, \operatorname{Im}\rho_{12}, \operatorname{Re}\rho_{13}, \operatorname{Im}\rho_{13}, ..., \rho_{44}]^T.$$

Thus, a two-qubit density matrix ρ is represented as a real vector $x = (x_1, ..., x_{16})$ with its elements given as follows

$$\rho(x) = \begin{bmatrix} x_1 & x_2 + ix_3 & x_4 + ix_5 & x_6 + ix_7\\ x_2 - ix_3 & x_8 & x_9 + ix_{10} & x_{11} + ix_{12}\\ x_4 - ix_5 & x_9 - ix_{10} & x_{13} & x_{14} + ix_{15}\\ x_6 - ix_7 & x_{11} - ix_{12} & x_{14} - ix_{15} & x_{16} \end{bmatrix}.$$

The coefficient matrices depend on the choice of the equations used for reconstructing a given density matrix. Below we list the transposed (for typographic reasons) coefficient matrices for the four analyzed tomographic protocols:

III. OBSERVATION VECTORS

The observation vectors correspond to photon coincidence counts. In reality we measure disturbed quantities $\bar{b} \equiv b + \delta b$ instead of b. The observation vectors are column vectors. For convenience we arrange them in arrays, where each column corresponds to one of the 17 reconstructed states.

$$\bar{b} = \begin{cases} \rho_1 & \rho_2 & \dots & \rho_{17} \\ \bar{b}_1 & \bar{b}_{1,1} & \bar{b}_{1,2} & \dots & \bar{b}_{1,17} \\ \bar{b}_{2,1} & \bar{b}_{2,2} & \dots & \bar{b}_{2,17} \\ \vdots & \vdots & \ddots & \vdots \\ \bar{b}_N & \bar{b}_{N,1} & \bar{b}_{N,2} & \dots & \bar{b}_{N,17} \end{bmatrix}$$

Note that the values of b listed below are not normalized and cannot be interpreted as probabilities. The elements of each vector b were registered over 5 seconds. This means that if an element of b is a sum or a difference of n projectors the measurement for each of the n projectors took 5/n seconds. In this way the measurements for observation vectors of the same length take the same amount of time. To obtain the frequencies we can divide these values by the total

number of photon coincidences counted or by a sum of coincidences counted for a set of projectors forming a basis. The set of such projectors is not unique. In our calculations we use the unnormalized coincidences and normalize the reconstructed density matrices.

A. Standard 36 state tomography

2727 2575 2844 1448 127 2762 3831 $1312 \ 2457$ 2773 3452 4102 2159 1096 1386 2126 1244 1401 1890 1461 1194 1001 1349 1010 2043 2310 1697 18651224 1120 $2102 \ 1407$ $1210 \ 1150$ 2409 2782 $1643 \ 1302$ $1581 \ 1216$ 2706 2747 2426 $1403 \ 1448 \ 1248$ $1440\ \ 1328\ \ 1144$ $2204 \ 1101$ $1798 \ 1108$ $1316 \ 1304$ 1250 1385 1231 1225 $1354 \ 1497$ $\bar{b}_S =$ 1266 1095 $1229 \ 1498$ 1173 1991 $1259 \ 1048$ 1348 2087 $1094 \ 1532$ 1133 2117 1994 2063 2345 1188 1343 1326 $1608 \ 1713 \ 1499 \ 2127 \ 1416$ 2354 828 $1028 \ 1672$ $1919\ 1103\ 1253$ 2470 2414 1691 1357 1323 1424 1634 2699 2757 1040 8521331 1459 1514 2563 1170 1148 1696 1372 1081 1267 2546 1817 1481 1784 1648 1243 2746 2140 1068 2106 2968 1278 1244 1040 3398 1327 1561 1413 1354 1335 1524 1198 $1327 \ 1275 \ 1066 \ 2657$ $1483 \ 1151 \ 1255$

The rows of the observation vector \bar{b}_S correspond to the following consecutive projectors: $|HH\rangle\langle HH|$, $|HV\rangle\langle HV|$, $|HD\rangle\langle HD|$, $|HA\rangle\langle HA|$, $|HL\rangle\langle HL|$, $|HR\rangle\langle HR|$, $|VH\rangle\langle VH|$, $|VV\rangle\langle VV|$, $|VD\rangle\langle VD|$, $|VA\rangle\langle VA|$, $|VL\rangle\langle VL|$, $|VR\rangle\langle VR|$, $|DH\rangle\langle DH|$, $|DV\rangle\langle DV|$, $|DD\rangle\langle DD|$, $|DA\rangle\langle DA|$, $|DL\rangle\langle DL|$, $|DR\rangle\langle DR|$, $|AH\rangle\langle AH|$, $|AV\rangle\langle AV|$, $|AD\rangle\langle AD|$, $|AA\rangle\langle AA|$, $|AL\rangle\langle AL|$, $|AR\rangle\langle AR|$, $|LH\rangle\langle LH|$, $|LV\rangle\langle LV|$, $|LD\rangle\langle LD|$, $|LA\rangle\langle LA|$, $|LL\rangle\langle LL|$, $|LR\rangle\langle LR|$, $|RH\rangle\langle RH|$, $|RV\rangle\langle RV|$, $|RD\rangle\langle RD|$, $|RA\rangle\langle RA|$, $|RL\rangle\langle RL|$, $|RR\rangle\langle RR|$.

	-2727	2575	2844	1448	127	193	25	2955	40	27	1264	809	1231	2762	3831	113	ך 112
	30	58	116	1312	2457	2364	3928	72	2555	2375	863	1263	757	2722	2773	3452	4102
	1244	1401	1890	2688	826	1483	2159	1096	1152	1386	2126	40	50	2251	4529	1758	2012
	1270	1697	1865	1224	1120	722	2102	1407	1433	1210	1150	1200	1057	5667	266	2409	2782
	126	43	61	648	2282	2660	13	23	1892	2199	1277	895	1141	977	347	1887	1295
	2706	2747	2426	1751	14	36	35	2274	8	26	928	1240	888	802	310	17	23
	1403	1448	1248	113	1051	1095	10	1134	951	1399	35	2079	2034	646	352	974	719
ī	940	1611	907	1238	1253	1422	11	1235	1018	1069	1099	756	1067	1769	9	852	664
$o_J =$	1691	1357	1323	92	1424	1634	31	1291	881	1002	2036	64	1885	2699	2757	1040	852
	1331	1459	1514	2563	1170	1148	1696	1372	1081	1267	23	2185	86	2546	1817	1481	1784
	1648	1243	2746	1707	30	1217	1114	42	991	2140	1068	946	939	2106	2968	1278	1244
	48	2814	1524	1198	1435	84	889	1204	2062	804	913	746	1349	5230	157	2610	2403
	1316	1304	1837	1250	1385	1781	42	1458	952	1172	1147	710	1251	901	1634	939	651
	1607	1537	981	1279	1149	920	1594	1156	1207	1065	991	1268	691	1085	987	1753	1986
	42	2845	1706	1231	1225	2556	962	1090	32	1482	996	1111	1012	790	1732	81	208
	1298	1459	2689	2352	2356	1818	564	43	876	156	47	72	1866	5	2458	1053	938

The rows of the observation vector \bar{b}_J correspond to the following consecutive projectors: $|HH\rangle\langle HH|$, $|HV\rangle\langle HV|$, $|HD\rangle\langle HD|$, $|HL\rangle\langle HL|$, $|VH\rangle\langle VH|$, $|VV\rangle\langle VV|$, $|VD\rangle\langle VD|$, $|VL\rangle\langle VL|$, $|RH\rangle\langle RH|$, $|RV\rangle\langle RV|$, $|RD\rangle\langle RD|$, $|RL\rangle\langle RL|$, $|DH\rangle\langle DH|$, $|DV\rangle\langle DV|$, $|DD\rangle\langle DD|$, $|DR\rangle\langle DR|$.

C. MUB-based tomography

952 1172 1147 1316 1304 1837 1250 1385 1781 710 1251 1607 1537 1279 1149 $1594 \ 1156 \ 1207 \ 1065$ 1266 1095 1014 1010 1231 1320 1593 1099 $1947 \ 1209$ 1265 1323 1327 1561 $\bar{b}_M =$ 1581 1216 1643 1302 $2121\ 1091$ 4213 2268 2022 1093 2693 1246 1404 1117 1920 1922 1652 3361 4078 1349 1156 1613 868

The rows of the observation vector \bar{b}_M correspond to the following consecutive projectors: $|DH\rangle\langle DH|$, $|DV\rangle\langle DV|$, $|AH\rangle\langle AH|$, $|AV\rangle\langle AV|$, $|LD\rangle\langle LD|$, $|LA\rangle\langle LA|$, $|RD\rangle\langle RD|$, $|RA\rangle\langle RA|$, $|VR\rangle\langle VR|$, $|VL\rangle\langle VL|$, $|HR\rangle\langle HR|$, $|HL\rangle\langle HL|$, $|\Phi^+\rangle\langle\Phi^+|$, $|\Phi^-\rangle\langle\Phi^-|$, $|\Psi^+\rangle\langle\Psi^+|$, $|\Psi^-\rangle\langle\Psi^-|$, $\frac{1}{2}(|DL\rangle+i|AR\rangle)(\langle DL|-i\langle AR|)$, $\frac{1}{2}(|DL\rangle-i|AR\rangle)(\langle DL|+i\langle AR|)$, $\frac{1}{2}(|DR\rangle+i|AL\rangle)(\langle DR|-i\langle AL|)$, $\frac{1}{2}(|DR\rangle-i|AL\rangle)(\langle DR|+i\langle AL|)$, where $|\Phi^{\pm}\rangle = (|HH\rangle \pm |VV\rangle)/\sqrt{2}$ and $|\Psi^{\pm}\rangle = (|HV\rangle \pm |VH\rangle)/\sqrt{2}$.

D. Optimal tomography

1	2727	2575	2844	1448	127	193	25	2955	40	27	1264	809	1231	2762	3831	113	112
	30	58	116	1312	2457	2364	3928	72	2555	2375	863	1263	757	2722	2773	3452	4102
	126	43	61	648	2282	2660	13	23	1892	2199	1277	895	1141	977	347	1887	1295
	2706	2747	2426	1751	14	36	35	2274	8	26	928	1240	888	802	310	17	23
	$\overline{108}$	103	444	1325	$\overline{473}$	263	223	$\overline{331}$	$\overline{98}$	188	1037	$\overline{1069}$	$\overline{947}$	$\overline{591}$	1219	$\overline{142}$	$\overline{149}$
	$\overline{155}$	240	241	$\overline{209}$	$\overline{91}$	$\overline{485}$	169	108	328	107	9	211	61	2823	$\overline{2980}$	709	766
	25	104	461	235	185	385	19	164	$\overline{12}$	13	$\overline{105}$	$\overline{196}$	61	$\overline{957}$	$\overline{543}$	$\overline{16}$	$\overline{18}$
ī	$\overline{231}$	70	131	1092	$\overline{356}$	$\overline{431}$	$\overline{14}$	163	$\overline{96}$	$\overline{11}$	$\overline{978}$	1039	$\overline{912}$	$\overline{763}$	$\overline{383}$	$\overline{219}$	$\overline{269}$
$b_O =$	$\overline{18}$	60	52	1045	$\overline{25}$	226	$\overline{19}$	$\overline{20}$	$\overline{30}$	215	1069	1008	999	$\overline{289}$	68	43	58
	$\overline{429}$	251	$\overline{297}$	144	28	12	11	117	20	$\overline{189}$	73	$\overline{348}$	$\overline{3}$	869	$\overline{296}$	$\overline{116}$	$\overline{27}$
	188	108	$\overline{252}$	$\overline{157}$	25	$\overline{151}$	176	$\overline{26}$	$\overline{29}$	129	57	31	$\overline{143}$	$\overline{702}$	$\overline{576}$	$\overline{81}$	$\overline{94}$
	$\overline{36}$	$\overline{205}$	$\overline{318}$	1202	182	121	412	284	247	40	1032	$\overline{1059}$	951	726	$\overline{142}$	498	545
	$\overline{62}$	13	11	540	884	2342	2	27	$\overline{2019}$	$\overline{73}$	$\overline{118}$	46	$\overline{73}$	$\overline{706}$	341	$\overline{1979}$	$\overline{1883}$
	1	$\overline{0}$	14	957	2060	393	151	7	$\overline{4}$	$\overline{2158}$	$\overline{978}$	$\overline{949}$	799	918	$\overline{634}$	52	$\overline{56}$
	$\overline{2660}$	2593	288	153	8	53	$\overline{28}$	$\overline{302}$	$\overline{58}$	$\overline{4}$	21	$\overline{68}$	176	821	$\overline{247}$	$\overline{127}$	$\overline{123}$
	48	$\overline{10}$	$\overline{2468}$	$\overline{1111}$	38	93	$\overline{11}$	2501	$\overline{18}$	$\overline{42}$	1153	1010	$\overline{1172}$	$\overline{482}$	390	6	$\overline{85}$

The negative elements of \bar{b}_O are marked with an overline. The rows of the observation vector \bar{b}_O correspond to the following consecutive measurements: $|HH\rangle\langle HH|$, $|HV\rangle\langle HV|$, $|VH\rangle\langle VH|$, $|VV\rangle\langle VV|$, $|HD\rangle\langle HD| - |HA\rangle\langle HA|$, $|HL\rangle\langle HL| - |HR\rangle\langle HR|$, $|DH\rangle\langle DH| - |AH\rangle\langle AH|$, $|LH\rangle\langle LH| - |RH\rangle\langle RH|$, $|VD\rangle\langle VD| - |VA\rangle\langle VA|$, $|VL\rangle\langle VL| - |VR\rangle\langle VR|$, $|DV\rangle\langle DV| - |AV\rangle\langle AV|$, $|LV\rangle\langle LV| - |RV\rangle\langle RV|$, $|\Psi^+\rangle\langle \Psi^+| - |\Psi^-\rangle\langle \Psi^-|$, $|\bar{\Psi}^+\rangle\langle \bar{\Psi}^+| - |\bar{\Psi}^-\rangle\langle \bar{\Psi}^-|$, $|\Phi^+\rangle\langle \Phi^+| - |\Phi^-\rangle\langle \Phi^-|$, $|\Phi^+\rangle\langle \bar{\Phi}^+| - |\bar{\Phi}^-\rangle\langle \bar{\Phi}^-|$, where $|\bar{\Phi}^\pm\rangle = (|HH\rangle \pm i|VV\rangle)/\sqrt{2}$ and $|\bar{\Psi}^\pm\rangle = (|HV\rangle \pm i|VH\rangle)/\sqrt{2}$.

E. Pauli matrices based tomography

 $\overline{57}$ $\overline{113}$ $\overline{52}$ $\overline{212}$ $\overline{126}$ $\overline{68}$ $\overline{986}$ $\overline{825}$ $\overline{63}$ $\overline{59}$ $\overline{2}$ $\overline{82}$ $\overline{79}$ $\overline{21}$ $\overline{24}$ $\overline{79}$ $\overline{216}$ $\overline{46}$ $\overline{132}$ $\overline{65}$ $\overline{23}$ $\overline{13}$ $\overline{74}$ $\overline{98}$ $\overline{269}$ $\overline{276}$ $\overline{214}$ $\overline{26}$ $\overline{19}$ $\overline{120}$ $\overline{359}$ $\overline{1005}$ $\overline{407}$ $\overline{68}$ $\overline{94}$ $\overline{55}$ $\overline{61}$ $\overline{263}$ $\overline{10}$ $\bar{b}_P =$ $\overline{224}$ $\overline{34}$ $\overline{973}$ $\overline{93}$ $\overline{45}$ $\overline{14}$ $\overline{104}$ $\overline{6}$ $\overline{60}$ $\overline{249}$ $\overline{1342}$ $\overline{113}$ $\overline{32}$ $\overline{27}$ $\overline{1100}$ $\overline{34}$ $\overline{20}$ $\overline{19}$ $\overline{39}$ $\overline{35}$ $\overline{16}$ $\overline{10}$ $\overline{249}$ $\overline{64}$ $\overline{65}$ $\overline{16}$ $\overline{31}$ $\overline{440}$ $\overline{50}$ $\overline{45}$ $\overline{292}$ $\overline{28}$ $\overline{33}$ $\overline{32}$ $\overline{237}$ $\overline{69}$ $\overline{47}$ $\overline{242}$ $\overline{16}$ $\overline{981}$ $\overline{44}$ $\overline{200}$ $\overline{367}$ $\overline{680}$ $\lfloor 1397 \ 1356 \ 1362 \ 1290 \ 1220 \ 1313 \ 1000 \ 1331 \ 1124 \ 1157 \ 1083 \ 1052 \ 1004 \ 1816 \ 1815 \ 1367 \ 1383 \ .$

The negative elements of \bar{b}_P are marked with an overline. The rows of the observation vector \bar{b}_P correspond to the following consecutive measurements: $(|DD\rangle\langle DD| + |AA\rangle\langle AA|) - (|DA\rangle\langle DA| + |AD\rangle\langle AD|)$, $(|DL\rangle\langle DL| + |AR\rangle\langle AR|) - (|DR\rangle\langle DR| + |AL\rangle\langle AL|)$, $(|DH\rangle\langle DH| + |AV\rangle\langle AV|) - (|DV\rangle\langle DV| + |AH\rangle\langle AH|)$, $(|DH\rangle\langle DH| + |AH\rangle\langle AH|) - (|DV\rangle\langle DV| + |AH\rangle\langle AH|)$, $(|DH\rangle\langle DH| + |AH\rangle\langle AH|) - (|DV\rangle\langle DV| + |AH\rangle\langle AH|)$, $(|DH\rangle\langle AV|)$, $(|LD\rangle\langle LD| + |RA\rangle\langle RA|) - (|LA\rangle\langle LA| + |RD\rangle\langle RD|)$, $(|LL\rangle\langle LL| + |RR\rangle\langle RR|) - (|LR\rangle\langle LR| + |RL\rangle\langle RL|)$, $(|LH\rangle\langle LH| + |RV\rangle\langle RV|) - (|LV\rangle\langle LV| + |RH\rangle\langle RH|)$, $(|LH\rangle\langle LH| + |RH\rangle\langle RH|) - (|LV\rangle\langle LV| + |RH\rangle\langle RH|)$, $(|LH\rangle\langle LH| + |RH\rangle\langle RH|) - (|LV\rangle\langle LV| + |RH\rangle\langle RH|)$, $(|HD\rangle\langle HL| + |VA\rangle\langle VA|) - (|HA\rangle\langle HA| + |VD\rangle\langle VD|)$, $(|HL\rangle\langle HL| + |VR\rangle\langle VR|) - (|HR\rangle\langle HR| + |VL\rangle\langle VL|)$, $(|HH\rangle\langle HH| + |VV\rangle\langle VV|) - (|HV\rangle\langle HV| + |VH\rangle\langle VH|)$, $(|HH\rangle\langle HH| + |VH\rangle\langle VH|) - (|HV\rangle\langle HV| + |VH\rangle\langle VH|)$, $(|HH\rangle\langle HH| + |VL\rangle\langle VL| - |VR\rangle\langle VR|)$, $(|HH\rangle\langle HH| - |HV\rangle\langle HV|) + (|VH\rangle\langle VH| - |VV\rangle\langle VV|)$, $(|HH\rangle\langle HH| + |HV\rangle\langle HV|) + (|VH\rangle\langle VV|)$.

IV. ERROR ANALYSIS

A. Estimated variances

For all the tomographies the vectors of variances for the 17 measured states are given as matrices

$$\sigma^{2}(b) = \begin{array}{cccc} \rho_{1} & \rho_{2} & \dots & \rho_{17} \\ b_{1} & \sigma^{2}(b_{1,1}) & \sigma^{2}(b_{1,2}) & \dots & \sigma^{2}(b_{1,17}) \\ \sigma^{2}(b_{2,1}) & \sigma^{2}(b_{2,2}) & \dots & \sigma^{2}(b_{2,17}) \\ \vdots & \vdots & \ddots & \vdots \\ \sigma^{2}(b_{N,1}) & \sigma^{2}(b_{N,2}) & \dots & \sigma^{2}(b_{N,17}) \end{array} \right].$$

They can be approximated directly with $\sigma^2(b) \approx \sigma^2(\bar{b}) = \bar{b}$ for all the tomographies except the optimal one and the tomography based on the Pauli matrices. For the optimal tomography the matrix of variances reads

$$\sigma^{2}(b_{O}) = \begin{bmatrix} 2727 & 2575 & 2844 & 1448 & 127 & 193 & 25 & 2955 & 40 & 27 & 1264 & 809 & 1231 & 2762 & 3831 & 113 & 112 \\ 30 & 58 & 116 & 1312 & 2457 & 2364 & 3928 & 72 & 2555 & 2375 & 863 & 1263 & 757 & 2722 & 2773 & 3452 & 4102 \\ 126 & 43 & 61 & 648 & 2282 & 2660 & 13 & 23 & 1892 & 2199 & 1277 & 895 & 1141 & 977 & 347 & 1887 & 1295 \\ 2706 & 2747 & 2426 & 1751 & 14 & 36 & 35 & 2274 & 8 & 26 & 928 & 1240 & 888 & 802 & 310 & 17 & 23 \\ 1352 & 1297 & 1445 & 1362 & 1299 & 1219 & 1936 & 1427 & 1250 & 1198 & 1088 & 1109 & 997 & 2842 & 3309 & 1900 & 2161 \\ 1425 & 1456 & 1624 & 1433 & 1211 & 1207 & 1933 & 1515 & 1104 & 1102 & 1141 & 989 & 995 & 2844 & 3246 & 1700 & 2016 \\ 1291 & 1199 & 1375 & 1015 & 1199 & 1395 & 23 & 1294 & 964 & 1158 & 1252 & 906 & 1190 & 1858 & 2177 & 955 & 669 \\ 1460 & 1427 & 1454 & 1184 & 1068 & 1203 & 16 & 1454 & 785 & 990 & 1058 & 1103 & 973 & 1936 & 2374 & 820 & 583 \\ 1421 & 1388 & 1196 & 1158 & 1076 & 1321 & 29 & 1154 & 981 & 1184 & 1104 & 1071 & 1035 & 935 & 284 & 930 & 660 \\ 1369 & 1359 & 1204 & 1094 & 1225 & 1410 & 22 & 1118 & 998 & 1258 & 1026 & 1104 & 1070 & 900 & 305 & 968 & 691 \\ 1419 & 1428 & 1233 & 1436 & 1124 & 1071 & 1770 & 1182 & 1236 & 1194 & 934 & 1236 & 834 & 1787 & 1563 & 1834 & 2080 \\ 1295 & 1253 & 1196 & 1360 & 1352 & 1269 & 2108 & 1088 & 1328 & 1307 & 1055 & 1125 & 1037 & 1820 & 1674 & 1979 & 2329 \\ 94 & 41 & 98 & 1018 & 2609 & 2596 & 1889 & 60 & 2194 & 2195 & 1209 & 821 & 1075 & 1533 & 1195 & 2565 & 2506 \\ 44 & 53 & 146 & 1065 & 2112 & 2196 & 1772 & 86 & 2074 & 2276 & 1031 & 1114 & 862 & 1685 & 1628 & 2274 & 2316 \\ 2757 & 2691 & 2431 & 1027 & 127 & 80 & 104 & 2640 & 74 & 27 & 1301 & 1026 & 1262 & 1778 & 2115 & 150 & 144 \\ 2806 & 2583 & 2521 & 1249 & 160 & 164 & 105 & 2580 & 55 & 67 & 1183 & 1049 & 1227 & 1737 & 1964 & 306 & 274 \end{bmatrix}$$

For the Pauli matrices based tomography the matrix of variances reads

$$\sigma^{2}(b_{P}) = \begin{bmatrix} 1323 & 1313 & 1259 & 1206 & 1165 & 1201 & 903 & 1240 & 1093 & 1163 & 1069 & 1052 & 1054 & 1927 & 1799 & 1380 & 1377 \\ 1326 & 1329 & 1327 & 1185 & 1142 & 1257 & 853 & 1254 & 1022 & 1189 & 1075 & 1036 & 1032 & 1886 & 1877 & 1384 & 1340 \\ 1355 & 1314 & 1304 & 1225 & 1162 & 1234 & 897 & 1238 & 1100 & 1176 & 1093 & 1072 & 1012 & 1823 & 1870 & 1395 & 1375 \\ 1355 & 1314 & 1304 & 1225 & 1162 & 1234 & 897 & 1238 & 1100 & 1176 & 1093 & 1072 & 1012 & 1823 & 1870 & 1395 & 1375 \\ 1392 & 1322 & 1323 & 1232 & 1228 & 1235 & 1089 & 1243 & 1089 & 1126 & 1042 & 1072 & 1016 & 1953 & 1952 & 1406 & 1455 \\ 1370 & 1320 & 1322 & 1230 & 1225 & 1225 & 1032 & 1311 & 1051 & 1090 & 1009 & 1062 & 1000 & 1863 & 1965 & 1396 & 1407 \\ 1378 & 1341 & 1325 & 1273 & 1210 & 1236 & 1062 & 1271 & 1057 & 1149 & 1056 & 1114 & 1005 & 1878 & 2024 & 1400 & 1456 \\ 1378 & 1341 & 1325 & 1273 & 1210 & 1236 & 1062 & 1271 & 1057 & 1149 & 1056 & 1114 & 1005 & 1878 & 2024 & 1400 & 1456 \\ 1378 & 1341 & 1325 & 1273 & 1210 & 1236 & 1062 & 1271 & 1057 & 1149 & 1056 & 1114 & 1005 & 1878 & 2024 & 1400 & 1456 \\ 1378 & 1341 & 1325 & 1273 & 1210 & 1236 & 1062 & 1271 & 1057 & 1149 & 1056 & 1114 & 1005 & 1878 & 2024 & 1400 & 1456 \\ 1378 & 1341 & 1325 & 1273 & 1210 & 1236 & 1062 & 1271 & 1057 & 1149 & 1056 & 1114 & 1005 & 1878 & 2024 & 1400 & 1456 \\ 1387 & 1343 & 1321 & 1261 & 1188 & 1270 & 983 & 1291 & 1116 & 1191 & 1097 & 1090 & 1016 & 1889 & 1797 & 1416 & 1411 \\ 1397 & 1408 & 1414 & 1264 & 1218 & 1309 & 978 & 1317 & 1051 & 1180 & 1084 & 1047 & 1033 & 1872 & 1776 & 1334 & 1354 \\ 1397 & 1356 & 1362 & 1290 & 1220 & 1313 & 1000 & 1331 & 1124 & 1157 & 1083 & 1052 & 1004 & 1816 & 1815 & 1367 & 1383 \\ 1387 & 1343 & 1321 & 1261 & 1188 & 1270 & 983 & 1291 & 1116 & 1191 & 1097 & 1090 & 1016 & 1889 & 1797 & 1416 & 1411 \\ 1397 & 1408 & 1414 & 1264 & 1218 & 1309 & 978 & 1317 & 1051 & 1180 & 1084 & 1047 & 1033 & 1872 & 1776 & 1334 & 1354 \\ 1397 & 1356 & 1362 & 1290 & 1220 & 1313 & 1000 & 1331 & 1124 & 1157 & 1083 & 1052 & 1004 & 1816 & 1815 & 1367 & 1383 \\ 1397 & 1356$$

B. Estimated error radii

For each tomography and reconstructed state we have estimated the maximum error R as described in the main text. Our results are summarized in the following matrix:

		Optimal	MUB	Standard	Pauli	JKMW
	ρ_1	0.0786	0.1180	0.1746	0.1926	0.4570
	ρ_2	0.0798	0.1243	0.1777	0.1958	0.4286
	ρ_3	0.0790	0.1205	0.1785	0.1938	0.4238
	ρ_4	0.0798	0.1182	0.1704	0.1843	0.4035
	ρ_5	0.0841	0.1298	0.1875	0.2053	0.4614
	ρ_6	0.0799	0.1271	0.1833	0.1959	0.4442
	ρ_7	0.0922	0.1647	0.2110	0.2578	0.4852
	ρ_8	0.0799	0.1264	0.1839	0.1964	0.4680
R =	ρ_9	0.0866	0.1370	0.2006	0.2146	0.4908
	ρ_{10}	0.0866	0.1306	0.1910	0.2134	0.4646
	ρ_{11}	0.0904	0.1221	0.1825	0.2009	0.4663
	ρ_{12}	0.0908	0.1278	0.1834	0.2050	0.4807
	ρ_{13}	0.0943	0.1256	0.1854	0.2105	0.4680
	ρ_{14}	0.0689	0.0992	0.1207	0.1635	0.3126
	ρ_{15}	0.0691	0.1025	0.1270	0.1702	0.3530
	ρ_{16}	0.0795	0.1218	0.1737	0.1983	0.4353
	ρ_{17}	0.0788	0.1250	0.1734	0.2025	0.4183

Note that the values are multiplied by a factor of 1.04 to compensate for underestimation of $\|\sigma\|$. Standard error is simply given by r = R/2.

It follows from the definition of R that $R \propto 1/\sqrt{t}$ is inverse proportional to square root of time t spent on a single tomographic measurement when all the photon counts are performed in equal 5 s time intervals. We would like to compare the tomographic protocols of various numbers of measurement setup settings N and fixed total amount of time spent on all the measurements. To do this with our measured data we postprocess the raw radii $R \to R_{\parallel} = sR$ by rescaling them by factors of $s = \sqrt{g}$, where $g = N/N_0$ and N_0 is the lowest number of settings for the fastest the protocol.

Using a setup with four bucket detectors allows to gather more precise data in the same time due to measurement paralellization. In this case the scaling factor reads $s = 3/\sqrt{7}$ for all the tomographies (all measurements can be taken using N = 9 settings) except the MUB tomography, where s = 1 ($N = N_0 = 7$ settings are enough). This means that in the four-detector regime the MUB tomography is 9/7 faster than others allowing gathering more precise data in shorter time. In order to incorporate this fact into our analysis we have multiplied the error radii for all tomographies except the MUB (s = 1) by a factor of $s = 3/\sqrt{7} \approx 1.134$, i.e.,

		Optimal	MUB	Standard	Pauli	JKMW
	ρ_1	[0.0892	0.1180	0.1980	0.2183	ך 0.5182
	ρ_2	0.0904	0.1243	0.2014	0.2221	0.4859
	ρ_3	0.0895	0.1205	0.2024	0.2197	0.4805
	ρ_4	0.0904	0.1182	0.1932	0.2090	0.4575
	ρ_5	0.0954	0.1298	0.2126	0.2328	0.5231
	$ ho_6$	0.0906	0.1271	0.2078	0.2222	0.5038
	ρ_7	0.1045	0.1647	0.2392	0.2922	0.5502
$R_{\parallel} =$	ρ_8	0.0906	0.1264	0.2086	0.2227	0.5306
	$ ho_9$	0.0982	0.1370	0.2274	0.2433	0.5565
	ρ_{10}	0.0982	0.1306	0.2166	0.2420	0.5268
	ρ_{11}	0.1025	0.1221	0.2069	0.2278	0.5287
	ρ_{12}	0.1030	0.1278	0.2080	0.2325	0.5451
	ρ_{13}	0.1070	0.1256	0.2102	0.2386	0.5306
	ρ_{14}	0.0781	0.0992	0.1369	0.1854	0.3544
	ρ_{15}	0.0784	0.1025	0.1439	0.1930	0.4002
	ρ_{16}	0.0902	0.1218	0.1970	0.2249	0.4936
	ρ_{17}	0.0894	0.1250	0.1966	0.2296	0.4743

but this does not change the fact that the smallest error is associated with the optimal tomography.

If there is no paralellization because only two detectors are used $(N_0 = 16)$, we obtain s = 1 for all the tomographies except the MUB $(s = \sqrt{20}/4)$ and standard tomography (s = 3/2). Thus, the two tomographies yield larger errors in comparison to the other tomographies due to the equal total time constraint. This is expected, because shorter time per measurement implies larger error per measurement. Thus, in this regime the optimal tomography is even better that it follows directly from our experimental data (estimated errors R).

C. Relative trace distances between the reconstructed states

In order to compare the quality of the matrices reconstructed with different tomographic protocols, we have also calculated the relative trace distances for the respective states in each protocols. Here we omitted the JKMW protocol, because it provides the largest error radius. Having three relative distances (for the remaining three protocols) it is possible to visualize the relative distances between the matrices and their error radii on a plane.

		$T(\rho_O, \rho_M)$	$T(\rho_O, \rho_S)$	$T(\rho_M, \rho_S)$
	ρ_1	Γ 0.1415	0.1004	0.1203
	ρ_2	0.1462	0.0798	0.1048
	ρ_3	0.1018	0.1130	0.1362
	ρ_4	0.1295	0.1786	0.1967
	$ ho_5$	0.0818	0.1684	0.1924
	$ ho_6$	0.1234	0.1176	0.0818
	ρ_7	0.0806	0.0483	0.0990
	$ ho_8$	0.1155	0.1150	0.1541
T =	$ ho_9$	0.1613	0.1245	0.0998
	ρ_{10}	0.1397	0.0956	0.1404
	ρ_{11}	0.0590	0.0515	0.0591
	ρ_{12}	0.1000	0.1074	0.0960
	ρ_{13}	0.0896	0.1000	0.0998
	ρ_{14}	0.0688	0.0425	0.0686
	ρ_{15}	0.0718	0.0790	0.0813
	ρ_{16}	0.1506	0.1270	0.1311
	ρ_{17}	0.1576	0.1278	0.1179





FIG. 4. Relative distances between points representing the reconstructed density matrices and their corresponding circles of the maximum errors R_{\parallel} optimal (O), standard (S), MUB-based (M) tomographies. The dashed circles correspond to the raw error radii R. The 12 states ρ_n are given in the units of trace distance. The states can be approximated by using Eq. (1) with $\rho_n = |\psi_n\rangle\langle\psi_n|$. The absolute positions of the three points are irrelevant. All the states for n = 1, ..., 12 are fully entangled except ρ_7 . The states of for n = 13, ..., 17 are partially entangled or separable (n = 13, 14). The ideally reconstructed state lies in the intersection of the error circles of radius R_{\parallel} (or R).