

Bell nonlocality and fully entangled fraction measured in an entanglement-swapping device without quantum state tomography

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We propose and experimentally implement an efficient procedure based on entanglement swapping to determine the Bell nonlocality measure of Horodecki *et al.* [*Phys. Lett. A* **200**, 340 (1995)] and the fully entangled fraction of Bennett *et al.* [*Phys. Rev. A* **54**, 3824 (1996)] of an arbitrary two-qubit polarization-encoded state. The nonlocality measure corresponds to the amount of the violation of the Clauser-Horne-Shimony-Holt (CHSH) optimized over all measurement settings. By using simultaneously two copies of a given state, we measure directly only six parameters. This is an experimental determination of these quantities without quantum state tomography or continuous monitoring of all measurement bases in the usual CHSH inequality tests. We analyze how well the measured degrees of Bell nonlocality and other entanglement witnesses (including the fully entangled fraction and a nonlinear entropic witness) of an arbitrary two-qubit state can estimate its entanglement. In particular, we measure these witnesses and estimate the negativity of various two-qubit Werner states. Our approach could especially be useful for quantum communication protocols based on entanglement swapping.

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Experimental methods for detecting and quantifying quantum entanglement [1,2] and Bell nonlocality (usually identified with the violation of a Bell inequality) [3,4] are of paramount importance for practical quantum information processing [5], quantum cryptography (e.g., quantum key distribution) [6], and quantum communication (e.g., quantum teleportation) [7,8]. Since the seminal experiments of Aspect *et al.* [9–11] in the early 1980's, various methods of detecting entanglement and nonlocality have been developed (for reviews, see Refs. [12,13]). Note that only very recently loophole-free tests of Bell nonlocality have been performed [14,15]. Nevertheless, measuring a degree of these effects seems to be much more difficult and important rather than only detecting them.

Thus, the question arises how to determine a measure of entanglement or nonlocality [12]. These can include the following for two qubits: (i) the negativity N [16,17], related to the Peres-Horodecki inseparability criterion [18,19], which is a measure of the entanglement cost under the operations preserving the positivity of the partial transpose of a state [20,21] and an estimator of the entanglement dimensionality, i.e., the number of the entangled degrees of freedom of two subsystems [22]; (ii) the concurrence C , corresponding to the entanglement of formation [23]; or (iii) the Bell nonlocality measure B of Refs. [24–27] corresponding the violation of the Bell inequality derived by Clauser, Horne, Shimony, and Holt (CHSH) [28], which is optimized over all measurements (i.e., detector settings) on the sides A (Alice) and B (Bob). We note that these measures are equivalent as $N = C = B$ for, e.g.,

phase-damped pure states (i.e., special kinds of Bell diagonal states) [26,29].

Here, we experimentally implement a direct and efficient method to conclusively detect and measure Bell nonlocality for a two-qubit mixed state without quantum state tomography (QST) and monitoring all measurement bases. In particular, for phase-damped two-qubit pure states, our method reduces to determining the concurrence and negativity. The presented approach is more efficient than QST, as we measure only partial information about a given state to measure nonlocality. Moreover, our detection of nonlocality is conclusive, in contrast to previous inconclusive detections of nonlocality witnesses without QST. We also measure another entanglement witness and estimator, i.e., the fully entangled fraction (FEF) f of a two-qubit state ρ , which is defined as [30] $f(\rho) = \max_{|e\rangle} \langle e|\rho|e\rangle$, where the maximum is taken over all maximally entangled states $|e\rangle$. The FEF is a useful concept in describing the fidelity of realistic quantum information protocols including dense coding, teleportation, entanglement swapping, quantum cryptography based on Bell's inequality, and, in general, multiqubit entanglement (see, e.g., Refs. [30–40]).

One could argue that the simplest experimental method for quantifying entanglement or nonlocality is to perform a complete QST to determine a given bipartite state ρ , and then to calculate (from ρ) the correlation measures related to a specific quantum information task. However, for the simplest case of two qubits in a general mixed state ρ , at least 15 measurements should be performed on identical copies of ρ to determine all 16 real parameters of ρ . Now the question arises whether a measure of quantum correlations could be determined directly or at least by a smaller number of measurements corresponding to an incomplete QST.

QST seems to be the easiest method of measuring entanglement or nonlocality. However, various alternative methods, for determining entanglement measures and nonlinear

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entanglement witnesses, have been attracting increasing interest [27,41–49]. Unfortunately, these methods (including those for a direct measurement of the Wootters concurrence [23]) use simultaneously at least two copies of a given state, which is not necessary for QST. Thus, one can raise the following objection: What is the advantage of using such direct methods? Indeed, these methods usually require a simultaneous manipulation with a few copies of a given state, while standard full QST can be based entirely on local measurements of a sequence of single qubits. There are reasons to look for alternative methods to QST: (i) For us, the most interesting and fundamental question is whether one can *directly* determine a nonlocality measure and estimate entanglement measures *without* knowing completely a given density matrix and monitoring all measurement bases. We show that it is possible both theoretically and experimentally. Clearly, QST collects more information than necessary to estimate a given quantum correlation quantity. Moreover, QST methods have some serious drawbacks, e.g.: (ii) The postprocessing of measured data in full QST of a state of N qubits scales exponentially with N . For example, Ref. [50] reported that experiments with eight qubits were performed within 8 h, while the postprocessing of the measured data for full QST took almost a week. (iii) Another fundamental problem is the nonphysicality of reconstructed states via standard QST. For example, Ref. [51] reported that a standard experimental QST of optical two-qubit states results in reconstructing nonphysical density matrices (i.e., with negative eigenvalues) about 75% of the time for low-entropy, highly entangled states. Then, a maximum-likelihood method or other techniques have to be applied to change such a nonphysical matrix into a physical one [51,52].

Various theoretical proposals to efficiently detect and quantify entanglement and nonlocality were described in, e.g., Refs. [27,41–47]. The first experimental direct measurement of a nonlinear entanglement witness was reported in Ref. [48], while the first experimental determination of an entanglement measure (i.e., the concurrence, being equal to the negativity and the CHSH measure) for a two-qubit pure state was reported in Ref. [49]. An experimental method for measuring a collective universal witness of any two-qubit mixed state (encoded in photon polarization) was proposed in Ref. [47]. It was later extended to probably the first method for measuring the negativity of an arbitrary two-qubit state [53,54]. All these theoretical and experimental studies show the fundamental difficulties not only in quantifying, but even in conclusively detecting, the entanglement or nonlocality of a two-qubit state without QST.

The CHSH inequality has been mostly used for detecting and quantifying [27] the Bell nonlocality of two qubits. This can be done by determining an optimal set of measurements for the correlated subsystems. If one deals with an unknown state, this approach requires applying all possible two-measurement settings for each qubit to find the optimal ones. However, as shown in Ref. [27], a more direct experimental procedure, which requires using only six detector settings, can be applied to find the maximal violation of the CHSH inequality for an arbitrary unknown two-qubit state. To avoid implementing inefficient procedures to be optimized over all possible measurement bases, this alternative approach of Ref. [27] requires simultaneously using two copies of a given two-qubit

state. The estimation of the amount of entanglement from the maximum violation of the CHSH inequality was studied in, e.g., Ref. [27].

We report here an experimental implementation of our six-step measurement procedure for determining the Bell nonlocality and fully entangled fraction with two copies of the investigated state and the singlet-state projection implemented by Hong-Ou-Mandel (anti)coalescence (see, e.g., Refs. [27,48,55–59]). In our experiment, we use polarization-encoded qubits. Our approach utilizes only a single two-photon interference event, instead of two required for standard nonlinear approaches [48,60]. However, we are able to measure the same nonlinear entropic entanglement witness, as in Refs. [48,60], for subsystems of equal purity.

The experimental complexity of our method of measuring the optimal CHSH inequality violation and the FEF is comparable to that of measuring the collectibility witness of Refs. [61–63] and can be implemented with the same experimental resources. Note that, contrary to the FEF and Bell nonlocality measure, the usefulness of the collectibility witness is limited mainly to pure or almost pure states only [61–63]. We apply the same method to measure both FEF and Bell nonlocality.

Entanglement swapping, which is a slightly generalized version of quantum teleportation, enables, together with single-qubit operations, universal quantum computation [64] and long-distance quantum communications [65,66]. Thus, this is not surprising that entanglement-swapping-type setups have been applied in many fundamental experiments (including the setup of Ref. [48] for measuring a nonlinear witness of entanglement without QST). This shows the universality of quantum information setups, which, after a minor modification, can be used for completely different purposes.

Theoretical framework. We study the entanglement and Bell nonlocality (corresponding to the CHSH inequality violation) of the polarization states of two photons [67]. The CHSH inequality for a two-qubit state $\rho \equiv \rho_{ab}$ can be written as [12,28] $|\text{Tr}(\rho \mathcal{B}_{\text{CHSH}})| \leq 2$. The maximum possible average value of the CHSH operator [24] is $\max_{\mathcal{B}_{\text{CHSH}}} |\text{Tr}(\rho \mathcal{B}_{\text{CHSH}})| = 2\sqrt{g(\rho)}$, where $\mathcal{B}_{\text{CHSH}} = \hat{a} \cdot \vec{\sigma} \otimes (\hat{b} + \hat{b}') \cdot \vec{\sigma} + \hat{a}' \cdot \vec{\sigma} \otimes (\hat{b} - \hat{b}') \cdot \vec{\sigma}$ depends on real unit vectors $\hat{a}, \hat{b}, \hat{a}', \hat{b}'$. The function $g(\rho) = \text{Tr} R - \min[\text{eig}(R)] \leq 2$ depends on the eigenvalues $\text{eig}(R)$ of a real symmetric matrix $R \equiv T^T T$, which is described with only six parameters, e.g., $R_{i,j}$ with $i \geq j$, where the correlation matrix T is defined as $T_{i,j} = \text{Tr}[\rho(\sigma_i \otimes \sigma_j)]$ for $i, j = 1, 2, 3$. As shown in Ref. [27], these six elements of R can be measured directly using two copies of ρ (i.e., ρ_1 and ρ_2). This is a consequence of the following identity, $R_{i,j} = \text{Tr}[(\rho_{a_1 b_1} \otimes \rho_{a_2 b_2}) S_{a_1 a_2} \otimes (\sigma_i \otimes \sigma_j)_{b_1 b_2}]$, where $\rho_{a_1 b_1} \equiv \rho_1$ and $\rho_{a_2 b_2} \equiv \rho_2$ for the subsystems a and b , whereas the operator $S_{a_1 a_2} = (I - 4|\Psi^-\rangle\langle\Psi^-|)_{a_1 a_2}$ is given in terms of the singlet state $|\Psi^-\rangle = (|HV\rangle - |VH\rangle)/\sqrt{2}$ and the two-qubit identity operation I , where H (V) stands for horizontal (vertical) polarization. The CHSH inequality $|\text{Tr}(\rho \mathcal{B}_{\text{CHSH}})| \leq 2$ is violated if $f(\rho) > 1/2$. Here, we apply the Bell nonlocality measure defined as [19] $M = g - 1 = \text{Tr} R - \min[\text{eig}(R)] - 1$, which is positive if and only if the CHSH inequality is violated and reaches its maximum $M = 1$ for maximally entangled

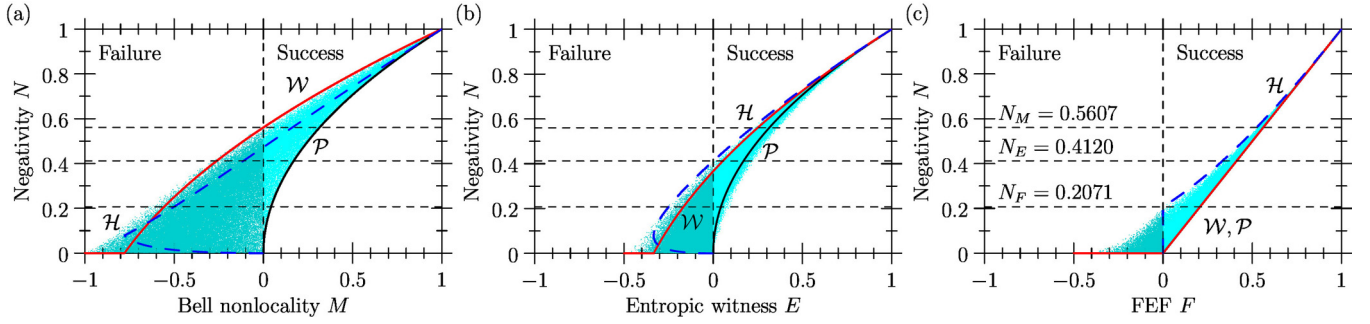


FIG. 1. Amount of entanglement measured with the negativity N vs (a) the Bell nonlocality M , (b) entropic witness E , and (c) fully entangled fraction F for 10^5 two-qubit states randomly generated by a Monte Carlo simulation. Entangled states for which an entanglement witness is successful in detecting inseparability are marked with light cyan dots. The entangled states that are ignored by the respective witness are marked with dark cyan dots. The Werner (\mathcal{W}), Horodecki (\mathcal{H}), and pure (\mathcal{P}) states correspond to the upper solid, dashed, and lower solid curves, respectively. In particular, F allows detecting the entanglement of the Werner states \mathcal{W} for the whole range of their mixing parameter $p > 1/3$. A given witness detects all the entangled states of the negativity above its respective threshold, i.e., $N_M = 0.5607$, $N_E = 0.4120$, and $N_F = 0.2071$.

states. Note that M is trivially related to $B = \sqrt{\max(M, 0)}$ studied in Refs. [26,27,29]. Moreover, we apply another entanglement witness, i.e., the (modified) FEF $F(\rho)$ [30] defined as $F = 2f - 1 = \frac{1}{2}(\text{Tr}\sqrt{R} - 1)$, which is a rescaled version of the standard FEF $f(\rho)$ [34,68,69]. Note that $F < 0$ for all separable states and is equal to the negativity for the Werner and pure states. These FEFs correspond to the fidelity of various entanglement-assisted processes maximized over all possible local unitary operations. The FEF

F detects more entangled states than both M and the nonlinear entropic witness $E = 2(\text{Tr} \rho_{ab}^2 - \min_{n=a,b} \text{Tr} \rho_n^2) = \frac{1}{2}(\text{Tr} R - 1 + |\text{Tr}(\rho_a^2 - \rho_b^2)|)$, which was measured in Ref. [48]. Note that $E = \frac{1}{2}(\text{Tr} R - 1)$ for $\text{Tr} \rho_a^2 = \text{Tr} \rho_b^2$. The spectrum of R , used in the definition of F , is measured unavoidably while measuring $M = g(\rho) - 1$, which quantifies the optimal CHSH violation. Thus, the optimal CHSH inequality is fundamentally more powerful in detecting quantum entanglement than its original form in an unoptimized measurement basis.

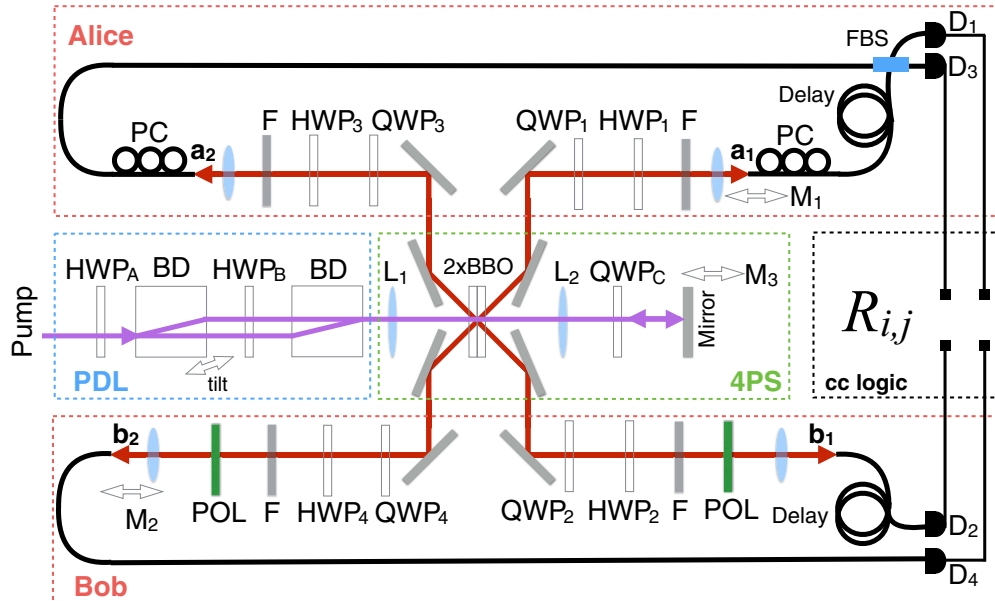


FIG. 2. Experimental scheme for determining the Bell nonlocality measure and fully entangled fraction of polarization-encoded two-qubit states with linear optics via the elements of the R matrix. This setup consists of narrow-band filters (F), half-wave plates (HWPs), quarter-wave plates (QWPs), beam dividers (BDs), detectors (D), a fiber beam splitter (FBS), polarization controllers (PCs), lenses (L), BBO crystals, mirrors, and motorized-translation stages (M). Note that this is an entanglement-swapping device, where the swapping is implemented by the FBS. The setup is powered by a laser system described in the text. The polarization-dispersion line (PDL) compensates the polarization dispersion introduced by the BBO crystals in the four-photon-source module (4PS). In this module, two copies of a two-qubit state ρ_1 and ρ_2 are prepared in the modes (a_1, b_1) and (a_2, b_2) , respectively. We name the last two modules as belonging to Alice and Bob, respectively. In Alice's module, qubit a_1 is overlapped on an 50:50 beam splitter with qubit a_2 to implement the measurement of $S_{a_1 a_2} = (I - 4|\Psi^-\rangle\langle\Psi^-|)_{a_1 a_2}$. In Bob's module, qubits b_1 and b_2 are projected onto the eigenstates of $(\sigma_i \otimes \sigma_j)_{b_1 b_2}$ for $i \geq j$ and $i, j = 1, 2, 3$ by the respective polarizer (POL) and detected at the respective detector. The fourfold coincidence counts are then processed to estimate the values of $R_{i,j}$.

The performance of a given entanglement witness can conveniently be studied with one-parameter ($0 \leq p \leq 1$) classes of states, including the Werner states $\mathcal{W}(p) = (1-p)|I/4 + p|\Psi^-\rangle\langle\Psi^-|$ [29,70], the Horodecki states $\mathcal{H}(p) = p|HH\rangle\langle HH| + (1-p)|\Psi^-\rangle\langle\Psi^-|$ [19], and pure states $\mathcal{P} = (\sqrt{p}|HH\rangle + \sqrt{1-p}|VV\rangle)(\text{H.c.})$. A comparison of the Bell nonlocality measure and entanglement witnesses for randomly generated states is presented in Fig. 1. These results show how well the negativity can be estimated by these entanglement witnesses.

Experiment. In our experiment we used a four-photon source shown in Fig. 2. This multiphoton source is pumped by a Coherent Mira femtosecond laser at a repetition rate of 80 MHz. The wavelength of the pulses is then converted in the process of second-harmonic generation (SHG) to 413 nm. On average, the mean power of the upconverted pumping beam is circa 300 mW. Next, the beam travels through a polarization-dispersion line (PDL) that compensates the polarization dispersion caused by the β -BaB₂O₄ crystals (BBO) used to create pairs of photons. The PDL was built by placing a half-wave plate (HWP) between two beam displacers (BDs). This construction allows us to tune the relative optical path of photons of selected polarization by tilting the BDs. The pumping beam then powers a BBO crystal cascade [71], which generates (in the process of type-I spontaneous parametric downconversion) pairs of horizontally and vertically polarized photons. The polarization and phase of a single photon pair can be adjusted by setting the correct polarization of the pumping beam. The beam passes through a quarter-wave plate (QWP) before and after being reflected by a mirror. This QWP compensates the polarization dispersion in the BBO crystals, which are now pumped in the opposite direction and create a second pair of photons.

The created pairs of photons are reflected by axillary mirrors to Alice and Bob who process the relevant photons (a_1, a_2) and (b_1, b_2) from each pair, respectively. The polarizations of photons b_1 and b_2 are first rotated by QWPs and HWPs and then projected by polarizers (POLs) to match an eigenstate of $(\sigma_m \otimes \sigma_n)_{b_1 b_2}$. Next, the photons are coupled to single-mode fibers and detected. Photons a_1 and a_2 are coupled to fibers directly, and then overlapped on a balanced fiber beam splitter (FBS) before being detected. Before entering the fibers photons a_1 and a_2 (b_1 and b_2) are filtered with 5 nm (10 nm) interference filters. Note that entanglement swapping in our setup can be implemented by the FBS.

The interference strength on the FBS is tuned by a proper choice of fiber delay and by setting the right position of the motorized-translation stage (M) associated with the corresponding mirror in a four-photon source (4PS). For its two extreme settings, the FBS performs the projective measurements $I/2$ or $|\Psi^-\rangle\langle\Psi^-|$. However, the optical couplers collect photon pairs generated at random distances from each other in the BBO crystal due to its group-velocity dispersion. Thus, a fraction of photons r will not overlap on the FBS, but can be detected in the same time window of the detectors as the perfectly overlapping photons, i.e., Alice performs the $[\frac{r}{2}I + (1-r)|\Psi^-\rangle\langle\Psi^-|]_{a_1 a_2}$ measurement. For each source configuration, we measure this fraction of noninteracting photons while calibrating the setup and setting the appropriate delays. Depending on the weight r , Alice performs a projection

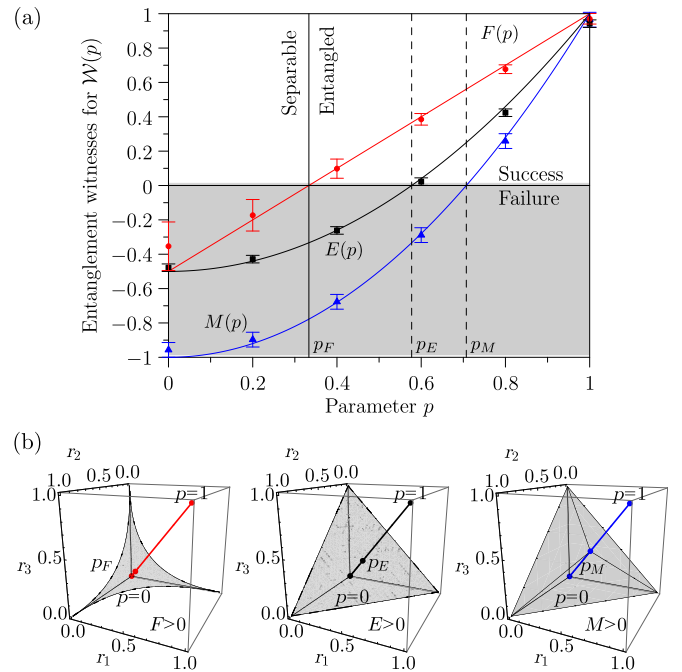


FIG. 3. Experimental detection of entanglement witnesses for the Werner states $\mathcal{W}(p)$: (a) Bell nonlocality M , fully entangled fraction F , and entropic witness E vs mixing parameter p . Our theoretical results for the perfect Werner states are marked with solid curves. The systematic deviation from the ideal case (solid curves) is caused by the fact that our experimentally created singlet state was not ideally pure and its purity reached circa 93%. Moreover, our experimentally created mixed state was not totally mixed. The ideal Werner states are separable for $p < 1/3$. For these states the entanglement can be detected with F , E , and M for $p_F > 1/3$, $p_E > 1/\sqrt{3}$, and $p_M > 1/\sqrt{2}$, respectively. In (b) we show the geometric representation of the witnesses. For each witness the separable states are enclosed by a respective gray boundary. The coordinate system is given as $[r_1, r_2, r_3] = \text{eig}(R)$. The Werner states for $0 \leq p \leq 1$ are located on the diagonals connecting points $[0, 0, 0]$ and $[1, 1, 1]$. The entanglement is detected for non-negative values of a witness, i.e., for points outside of the gray areas. For F (the most universal of the three witnesses) this area is the smallest.

on a particular Werner state. Thus, the uncertainty of the obtained results is limited only by the number of registered coincidences and the precision of determining the weight r . In this setup, we typically register one fourfold coincidence event in 5 min. We collected hundreds of such coincidences per measurement setting. In our experiment we experimentally studied two kinds of two-qubit states, namely, the pure separable states $\mathcal{P}(0) = |VV\rangle\langle VV|$, and the Werner states, which can be entangled even if $M < 0$ (see Fig. 3). In particular, we measured the maximally entangled states $\mathcal{W}(1)$ and the completely mixed state $\mathcal{W}(0)$. These states were prepared using the method described in Ref. [63]. The Werner states are particularly important for quantum technologies because entanglement purification schemes transform other states into the Werner states (see Ref. [72] and references therein). Our measurement results for the Werner states are summarized in Fig. 3 and Table I. In all these cases, we reconstructed matrices R and applied the maximum-likelihood method to

TABLE I. Experimentally and theoretically obtained values of the Bell nonlocality M , entropic witness E , and fully entangled fraction F .

Density matrix	$M^{(\text{experiment})}$	$E^{(\text{experiment})}$	$F^{(\text{experiment})}$	$M^{(\text{theory})}$	$E^{(\text{theory})}$	$F^{(\text{theory})}$
$ \Psi^-\rangle\langle\Psi^- $	$+0.965 \pm 0.043$	$+0.942 \pm 0.022$	$+0.970 \pm 0.031$	+1	+1	+1
$ VV\rangle\langle VV $	-0.002 ± 0.043	-0.001 ± 0.022	-0.001 ± 0.08	0	0	0
$I/4$	-0.957 ± 0.043	-0.478 ± 0.022	-0.353 ± 0.141	-1	$-\frac{1}{2}$	$-\frac{1}{2}$

estimate their spectra. For the remaining experimental results and technical details, see the Supplemental Material [67].

Conclusions. We reported an experimental method for determining the Bell nonlocality measure [24] and fully entangled fraction [30] without quantum state tomography or scanning of all measurement bases. By applying the maximum-likelihood method, we demonstrated that direct measurements of nonlinear entanglement witnesses [47,48] could be made robust to experimental errors by exploiting the correlations between them. Our procedure is applicable if the mean experimental matrix $R^{(\text{exp})}$ can be well approximated with its maximum-likelihood estimate R [67]. This method could further be applied to improve the error robustness of entanglement measures [53].

We measured the Bell nonlocality by means of two-photon pairs prepared in the same state and six independent measurements in our entanglement-swapping device. In the orthodox CHSH approach, Alice and Bob perform two measurements on two copies of a given two-qubit state (four measurements in total). However, to determine the Bell nonlocality for an unknown state, they need to perform full quantum state tomography or to optimize their measurement bases, which requires performing four measurements in each optimization step, resulting in many more measurements than in our experiment.

Moreover, we demonstrated that the fully entangled fraction F is a better estimator of entanglement than the Bell nonlocality M and nonlinear entropic witness E , which was first measured in Ref. [48] via the double

Hong-Ou-Mandel interference. To show this, we measured the Werner states, which are recognized to be entangled by a particular entanglement witness if they have a large enough value of the mixing parameter p (see Fig. 3). For F , this critical value is $p_F > \frac{1}{3}$, which corresponds to the range for which the Werner states are entangled. For E and M , the entanglement of the Werner states occurs if $p_E > 1/\sqrt{3}$ and $p_M > 1/\sqrt{2}$, respectively.

Our method solves the problem of detecting and quantifying entanglement beyond a simple Bell test in a typical entanglement-swapping method [73], which can be applied to, e.g., quantum repeaters [74] and quantum relays [75] in device-independent quantum communications [76], as well as to entanglement-assisted quantum error correction [77] and entanglement purification [72].

We hope that our results could stimulate further research on measuring such nonlinear properties of quantum systems as entanglement and nonlocality without performing full quantum state tomography.

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