

# Noise-Tolerant Optomechanical Entanglement via Synthetic Magnetism

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Entanglement of light and multiple vibrations is a key resource for multi-channel quantum information processing and memory. However, entanglement generation is generally suppressed, or even fully destroyed, by dark modes formed by multiple vibrational modes coupled to a common optical mode. Here we propose how to generate both *bipartite* and *genuine tripartite* optomechanical entanglement via *dark-mode* (DM) *breaking* induced by synthetic magnetism. We find that at nonzero temperature, light and vibrations are *separable* in the DM-unbreaking regime but strongly *entangled* in the DM-breaking regime. Remarkably, its noise tolerance (the threshold thermal phonon number for preserving entanglement) is up to *three* orders of magnitude stronger than that in the DM-unbreaking regime. The application of the DM-breaking mechanism to optomechanical networks can make noise-tolerant entanglement networks feasible. These results are quite general and can initiate advances in quantum resources with immunity against both dark modes and thermal noise.

*Introduction.*—Quantum entanglement [1], allowing for inseparable quantum correlations shared by distant parties, is a crucial resource for modern quantum technologies, including quantum metrology, communication, and computation [2]. So far, efficient entanglement of photons with atoms [3–9], trapped ions [10, 11], quantum dots [12], and superconduction qubits [13–15] has been demonstrated in both microscopic- and macroscopic-scale devices [16, 17]. These entangled states have been used to connect remote long-term memory nodes in distributed quantum networks [18–21].

The cavity optomechanical system is an elegant candidate for implementing quantum information carrier and memory [22–24]. Owing to the remarkable progresses in ground-state cooling [25–28] and single-phonon manipulation [29–32], it has become a more efficient platform for achieving strong entanglement between two bosonic modes [33–47]. In particular, macroscopic quantum entanglement involving two massive mechanical oscillators has recently been observed in optomechanical platforms [48–51]. Practically, the applicability of modern quantum technologies in optomechanical networks ultimately requires quantum entanglement of light and multiple vibrations [52–55]. Realization of large-scale photon-phonon entanglement, however, remains an outstanding challenge due to the suppression from the dark modes [56, 57] induced by the coupling of multiple vibrational modes to a common optical mode [58–66].

In this Letter, we propose to generate strong light-vibration entanglement by breaking the dark mode via synthetic magnetism, and reveal its *counterintuitive robustness* to thermal noise. By introducing a loop-coupled structure,

formed by light-vibration couplings and phase-dependent phonon-hopping interactions, a synthetic gauge field is induced, and it breaks dark modes. Note that the realization of a reconfigurable synthetic gauge field has recently been reported in phase-dependent loop-coupled optomechanical platforms [67–75]. We find that, in the dark-mode-unbreaking (DMU) regime, both bipartite and genuine tripartite optomechanical entanglement are destroyed by thermal noise concealed in the dark modes; while in the dark-mode-breaking (DMB) regime, strong entanglement is generated via synthetic magnetism. Importantly, the DMB entanglement is surprisingly noise tolerant, in the sense that the threshold thermal phonon number for preserving entanglement is up to *three* orders of magnitude stronger than that in the DMU regime. Our work describes a general mechanism, and it can provide the means to engineer and protect fragile quantum resources from thermal noises and dark modes, and pave a way towards noise-tolerant quantum networks [18, 55].

*System and dark-mode control.*—We consider a loop-coupled optomechanical system consisting of an optical mode and two vibrational modes [see Fig. 1(a)]. To induce a synthetic magnetism, we introduce a phase-dependent phonon-hopping interaction between the two vibrations (with coupling strength  $\chi$  and modulation phase  $\Theta$ ). A driving field, with frequency  $\omega_L$  and amplitude  $\Omega = \sqrt{2\kappa\mathcal{P}_L}/(\hbar\omega_L)$  (for driving laser power  $\mathcal{P}_L$  and optical decay rate  $\kappa$ ), is applied to the cavity field. In a rotating frame defined by the unitary transformation operator  $\exp(-i\omega_L c^\dagger ct)$ , the system Hamiltonian reads

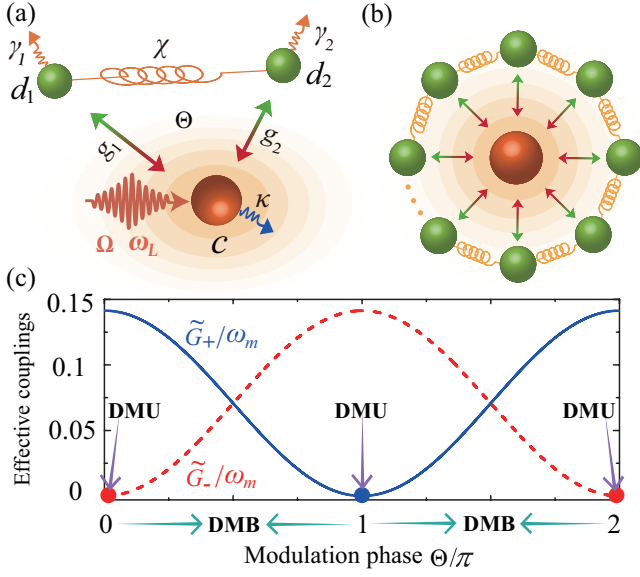


FIG. 1. (a) A loop-coupled optomechanical system consisting of two vibrational modes  $d_{j=1,2}$  (with decay rates  $\gamma_j$ ) coupled to a common optical mode  $c$  (with decay rate  $\kappa$ ) via radiation-pressure interactions (with strengths  $g_j$ ). The two vibrations are coupled to each other through a phase-dependent phonon-exchange coupling ( $\chi$  and  $\Theta$ ). (b) An optomechanical network consisting of an optical mode coupled to  $N$  vibrational modes. (c) Effective coupling strengths  $\tilde{G}_\pm$  versus the modulation phase  $\Theta$ . By tuning  $\Theta \neq n\pi$  for an integer  $n$ , the dark mode can be broken ( $\tilde{G}_\pm \neq 0$ ). The solid disks denote the DMU regime, and the remaining areas correspond to the DMB regime. Here, the parameters are set as  $\omega_j = \omega_m$  and  $G_j/\omega_m = \chi/\omega_m = 0.1$ .

( $\hbar = 1$ )

$$\begin{aligned} \mathcal{H}_I &= \Delta_c c^\dagger c + \sum_{j=1,2} [\omega_j d_j^\dagger d_j + g_j c^\dagger c (d_j + d_j^\dagger)] \\ &\quad + (\Omega c + \Omega^* c^\dagger) + \mathcal{H}_\chi, \\ \mathcal{H}_\chi &= \chi (e^{i\Theta} d_1^\dagger d_2 + e^{-i\Theta} d_2^\dagger d_1), \end{aligned} \quad (1)$$

where  $c^\dagger$  ( $c$ ) and  $d_j^\dagger$  ( $d_j$ ) are the creation (annihilation) operators of the cavity-field mode (with resonance frequency  $\omega_c$ ) and the  $j$ th vibrational mode (with resonance frequency  $\omega_j$ ), respectively. The  $g_j$  terms describe optomechanical interactions between the cavity-field mode and the two vibrational modes, with  $g_j$  being the single-photon optomechanical-coupling strength. The  $\Omega$  term denotes the cavity-field driving with detuning  $\Delta_c = \omega_c - \omega_L$ . The  $\mathcal{H}_\chi$  term depicts the phase-dependent phonon-hopping interaction, which is introduced to create synthetic gauge fields and control the dark-mode effect.

To demonstrate the dark-mode effect, we expand the operators  $o \in \{c, d_j, c^\dagger, d_j^\dagger\}$  as a sum of their steady-state average values and fluctuations, i.e.,  $o = \langle o \rangle_{ss} + \delta o$ . Then we obtain the linearized Hamiltonian in the rotating-wave approximation (RWA) as:  $\mathcal{H}_{\text{RWA}} = \Delta \delta c^\dagger \delta c + \sum_{j=1,2} [\omega_j \delta d_j^\dagger \delta d_j + G_j (\delta c \delta d_j^\dagger + \text{H.c.})] + \chi (e^{i\Theta} \delta d_1^\dagger \delta d_2 +$

H.c.), where  $\Delta$  is the normalized driving detuning and  $G_{j=1,2} = g_j \langle c \rangle_{ss}$  are the linearized optomechanical-coupling strengths. Here  $\langle c \rangle_{ss} = -i\Omega^*/(\kappa + i\Delta)$  is assumed to be real by choosing a proper  $\Omega$ .

When the synthetic magnetism is absent (i.e.,  $\chi = 0$ ) and  $\omega_1 = \omega_2$ , the system possesses two hybrid mechanical modes: a *bright mode* ( $\mathcal{D}_+$ ) and a *dark mode* ( $\mathcal{D}_-$ ) defined by

$$\mathcal{D}_+ = (G_1 \delta d_1 + G_2 \delta d_2) / \sqrt{G_1^2 + G_2^2}, \quad \text{Bright} \quad (2a)$$

$$\mathcal{D}_- = (G_2 \delta d_1 - G_1 \delta d_2) / \sqrt{G_1^2 + G_2^2}, \quad \text{Dark} \quad (2b)$$

which satisfy the bosonic commutation relation  $[\mathcal{D}_\pm, \mathcal{D}_\pm^\dagger] = 1$ . The dark mode  $\mathcal{D}_-$ , which decouples from the system and destroys all quantum resources, can be broken by employing the synthetic magnetism (i.e.,  $\chi \neq 0$  and  $\Theta \neq 0$ ). To clarify this, we introduce two superposition-vibrational modes associated with the synthetic magnetism:  $\tilde{\mathcal{D}}_+ = \mathcal{F} \delta d_1 - e^{i\Theta} \mathcal{K} \delta d_2$  and  $\tilde{\mathcal{D}}_- = e^{-i\Theta} \mathcal{K} \delta d_1 + \mathcal{F} \delta d_2$ , where  $\mathcal{F} = |\delta \tilde{\omega}_-| / \sqrt{(\delta \tilde{\omega}_-)^2 + \chi^2}$  and  $\mathcal{K} = \chi \mathcal{F} / \delta \tilde{\omega}_-$ , with  $\delta \tilde{\omega}_- = \tilde{\omega}_- - \omega_1$  and the redefined resonance frequencies  $\tilde{\omega}_\pm = (\omega_1 + \omega_2 \pm \sqrt{(\omega_1 - \omega_2)^2 + 4\chi^2})/2$ . The linearized Hamiltonian becomes

$$\mathcal{H}_{\text{RWA}} = \Delta \delta c^\dagger \delta c + \sum_{l=\pm} [\tilde{\omega}_l \tilde{\mathcal{D}}_l^\dagger \tilde{\mathcal{D}}_l + (\tilde{G}_l \tilde{\mathcal{D}}_l \delta c^\dagger + \text{H.c.})], \quad (3)$$

where the effective coupling strengths are given by  $\tilde{G}_+ = \mathcal{F} G_1 - e^{-i\Theta} \mathcal{K} G_2$  and  $\tilde{G}_- = e^{i\Theta} \mathcal{K} G_1 + \mathcal{F} G_2$ .

In Fig. 1(c), we show  $\tilde{G}_\pm$  versus  $\Theta$  when  $\omega_1 = \omega_2$  and  $G_1 = G_2$ . We can see that only when  $\Theta = n\pi$  (i.e., the DMU regime), either  $\tilde{\mathcal{D}}_+$  [for an odd  $n$ ,  $\tilde{G}_+ = 0$  (blue disks)] or  $\tilde{\mathcal{D}}_-$  [for an even  $n$ ,  $\tilde{G}_- = 0$  (red disks)] becomes a dark mode. Tuning  $\Theta \neq n\pi$  (for an integer  $n$ , i.e., the DMB regime) leads to a counterintuitive coupling of the dark mode to the optical mode, which indicates dark-mode breaking. Physically, a reconfigurable synthetic gauge field is realized by modulating the phase  $\Theta$ , which results in a flexible switch between the DMB and DMU regimes.

*The Langevin equations and their solutions.*— By defining the optical and mechanical quadratures  $\delta X_o = (\delta o^\dagger + \delta o) / \sqrt{2}$  and  $\delta Y_o = i(\delta o^\dagger - \delta o) / \sqrt{2}$ , and the corresponding Hermitian input-noise operators  $X_o^{\text{in}} = (o_{\text{in}}^\dagger + o_{\text{in}}) / \sqrt{2}$  and  $Y_o^{\text{in}} = i(o_{\text{in}}^\dagger - o_{\text{in}}) / \sqrt{2}$ , we obtain the linearized Langevin equations as  $\dot{\mathbf{u}}(t) = \mathbf{A} \mathbf{u}(t) + \mathbf{N}(t)$ , where we introduce the fluctuation operator vector  $\mathbf{u}(t) = [\delta X_{d_1}, \delta Y_{d_1}, \delta X_{d_2}, \delta Y_{d_2}, \delta X_c, \delta Y_c]^T$ , the noise operator vector  $\mathbf{N}(t) = \sqrt{2} [\sqrt{\gamma_1} X_{d_1}^{\text{in}}, \sqrt{\gamma_1} Y_{d_1}^{\text{in}}, \sqrt{\gamma_2} X_{d_2}^{\text{in}}, \sqrt{\gamma_2} Y_{d_2}^{\text{in}}, \sqrt{\kappa} X_c^{\text{in}}, \sqrt{\kappa} Y_c^{\text{in}}]^T$ ,

and the coefficient matrix

$$\mathbf{A} = \begin{pmatrix} -\gamma_1 & \omega_1 & \chi_+ & \chi_- & 0 & 0 \\ -\omega_1 & -\gamma_1 & -\chi_- & \chi_+ & -2G_1 & 0 \\ -\chi_+ & \chi_- & -\gamma_2 & \omega_2 & 0 & 0 \\ -\chi_- & -\chi_+ & -\omega_2 & -\gamma_2 & -2G_2 & 0 \\ 0 & 0 & 0 & 0 & -\kappa & \Delta \\ -2G_1 & 0 & -2G_2 & 0 & -\Delta & -\kappa \end{pmatrix}, \quad (4)$$

with  $\chi_+ = \chi \sin \Theta$  and  $\chi_- = \chi \cos \Theta$ . The formal solution of the Langevin equation is given by  $\mathbf{u}(t) = \mathbf{M}(t)\mathbf{u}(0) + \int_0^t \mathbf{M}(t-s)\mathbf{N}(s)ds$ , where  $\mathbf{M}(t) = \exp(\mathbf{A}t)$ . Note that the parameters used in our simulations satisfy the stability conditions derived from the Routh-Hurwitz criterion [76]. The steady-state properties of the system can be inferred based on the steady-state covariance matrix  $\mathbf{V}$ , which is defined by the matrix elements  $V_{kl} = [\langle \mathbf{u}_k(\infty)\mathbf{u}_l(\infty) \rangle + \langle \mathbf{u}_l(\infty)\mathbf{u}_k(\infty) \rangle]/2$  for  $k, l = 1-6$ . Under the stability conditions, the covariance matrix  $\mathbf{V}$  fulfills the Lyapunov equation  $\mathbf{A}\mathbf{V} + \mathbf{V}\mathbf{A}^T = -\mathbf{Q}$  [33], where  $\mathbf{Q} = \text{diag}\{\gamma_1(2\bar{n}_1 + 1), \gamma_1(2\bar{n}_1 + 1), \gamma_2(2\bar{n}_2 + 1), \gamma_2(2\bar{n}_2 + 1), \kappa, \kappa\}$ .

*Generating bipartite and genuine tripartite entanglements via dark-mode breaking.*—The logarithmic negativity  $E_{\mathcal{N},j}$  and the *minimum* residual contangle  $E_{\tau}^{r|s|t}$  [77–80], which can be used to quantify bipartite and genuine tripartite entanglement, are, respectively, defined as

$$E_{\mathcal{N},j} \equiv \max[0, -\ln(2\zeta_j^-)], \quad (5a)$$

$$E_{\tau}^{r|s|t} \equiv \min_{(r,s,t)} [E_{\tau}^{r|(st)} - E_{\tau}^{r|s} - E_{\tau}^{r|t}]. \quad (5b)$$

Here  $\zeta_j^- \equiv 2^{-1/2} \{ \Sigma(\mathbf{V}'_j) - [\Sigma(\mathbf{V}'_j)^2 - 4\det\mathbf{V}'_j]^{1/2} \}^{1/2}$ , with  $\Sigma(\mathbf{V}'_j) \equiv \det\mathbf{A}_j + \det\mathbf{B} - 2\det\mathbf{C}_j$ , is the smallest eigenvalue of the partial transpose of the reduced correlation matrix  $\mathbf{V}'_j = \begin{pmatrix} \mathbf{A}_j & \mathbf{C}_j \\ \mathbf{C}_j^T & \mathbf{B} \end{pmatrix}$ , which is obtained by removing in  $\mathbf{V}$  the rows and columns of the uninteresting mode [81]. In Eq. (5b),  $(r, s, t) \in \{d_1, d_2, c\}$  denotes all the permutations of the three mode indices.  $E_{\tau}^{r|(st)}$  ( $E_{\tau}^{r|s}$  or  $E_{\tau}^{r|t}$ ) is the contangle of subsystems of  $r$  and  $st$  ( $s$  or  $t$ ), and it is defined as the squared logarithmic negativity [78, 81]. The residual contangle satisfies the monogamy of quantum entanglement  $E_{\tau}^{r|(st)} \geq E_{\tau}^{r|s} + E_{\tau}^{r|t}$ , which is based on the Coffman-Kundu-Wootters monogamy inequality [80].  $E_{\mathcal{N},j} > 0$  and  $E_{\tau}^{r|s|t} > 0$  mean, respectively, the emergence of bipartite and *genuine* tripartite entanglement.

We display in Figs. 2(a) and 2(b) the bipartite entanglement  $E_{\mathcal{N},j}$  of the optical mode with the  $j$ th vibration and the tripartite entanglement  $E_{\tau}^{r|s|t}$  of the system as functions of the driving detuning  $\Delta$ , when the system operates in both DMU and DMB regimes. This demonstrates that light and vibrations are separable in the DMU regime ( $E_{\mathcal{N},j} = E_{\tau}^{r|s|t} = 0$ , see the lower horizontal solid lines), but strongly entangled in the DMB regime [ $E_{\mathcal{N},1(2)} = 0.14$  (0.12) and  $E_{\tau}^{r|s|t} = 0.013$ , see the upper dashed curves and symbols]. In

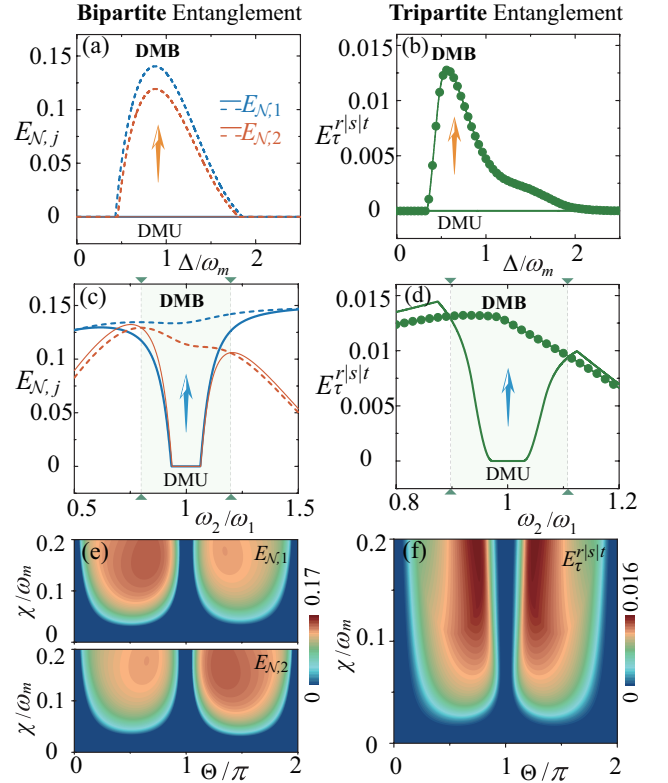


FIG. 2. (a) Bipartite entanglement measure  $E_{\mathcal{N},j=1,2}$  and (b) genuine tripartite entanglement measure  $E_{\tau}^{r|s|t}$  versus  $\Delta/\omega_m$  in the DMU ( $\chi = 0$ , horizontal solid lines) and DMB ( $\chi = 0.1\omega_m$  and  $\Theta = \pi/2$ , dashed curves and symbols) regimes. Here we take  $\omega_1 = \omega_2 = \omega_m$ . (c)  $E_{\mathcal{N},j}$  and (d)  $E_{\tau}^{r|s|t}$  versus  $\omega_2/\omega_1$  in both DMU and DMB regimes under the optimal drivings  $\Delta = \omega_1$  for  $E_{\mathcal{N},j}$  and  $\Delta = 0.6\omega_1$  for  $E_{\tau}^{r|s|t}$ . (e)  $E_{\mathcal{N},j}$  and (f)  $E_{\tau}^{r|s|t}$  versus  $\chi$  and  $\Theta$ . Other parameters are  $G_j/\omega_m = 0.2$ ,  $\gamma_j/\omega_m = 10^{-5}$ ,  $\kappa/\omega_m = 0.2$ , and  $\bar{n}_j = 100$ .

the DMU regime, thermal phonons concealed in the dark mode cannot be extracted by the optomechanical cooling channel, and then quantum entanglement is completely destroyed by the residual thermal noise [35, 58, 63]. In the DMB regime, a large entanglement is achieved around the red-sideband resonance ( $\Delta \approx \omega_m$ ), corresponding to the optimal cooling. These results indicate that *fragile quantum resources can be protected via dark-mode control*.

The dependence of quantum entanglement on the dark-mode effect can also be seen by plotting  $E_{\mathcal{N},j}$  and  $E_{\tau}^{r|s|t}$  as functions of the frequency ratio  $\omega_2/\omega_1$  [see Figs. 2(c) and 2(d)]. We find that in the DMU regime, there exists a disentanglement valley around the degeneracy point  $\omega_2 = \omega_1$ , corresponding to the emergence of the dark-mode effect (see the lower solid curves). However, in the DMB regime, the valley becomes smooth, which means that both bipartite and *genuine* tripartite entanglement fully survive owing to dark-mode breaking (see the upper dashed curves and symbols). Physically, the width of the valley

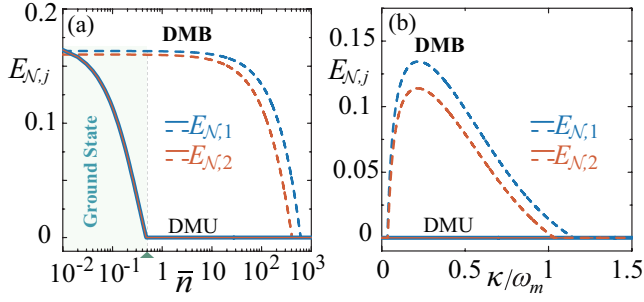


FIG. 3. (a)  $E_{N,j=1,2}$  versus  $\bar{n}_j = \bar{n}$  in the DMU (solid curves) and DMB (dashed curves) regimes. (b)  $E_{N,j}$  versus  $\kappa$  when  $\bar{n} = 100$ . Here we take  $\chi = 0.1\omega_m$  and  $\Theta = \pi/2$ , and other parameters are the same as those in Fig. 2.

is determined by the spectral resolution in the three-mode system. Here, it is mainly determined by the cavity-field decay rate, because the decay rates of the mechanical modes are much smaller than that of the cavity field.

In Figs. 2(e) and 2(f), we plot  $E_{N,j}$  and  $E_\tau^{r|s|t}$  versus  $\Theta$  and  $\chi$ . We can see that, at a finite value of  $\chi$ ,  $E_{N,j}$  and  $E_\tau^{r|s|t}$  reach the peak value at  $\Theta = \pi/2$  and  $3\pi/2$ , which are related to the maximal quantum interference effect. In addition, both bipartite and tripartite entanglement completely vanish, i.e.,  $E_{N,j} = 0$  and  $E_\tau^{r|s|t} = 0$  at  $\Theta = n\pi$ , corresponding to the emergence of the dark mode. In particular,  $E_{N,1}$  ( $E_{N,2}$ ) is larger than  $E_{N,2}$  ( $E_{N,1}$ ) in the region  $0 < \Theta < \pi$  ( $\pi < \Theta < 2\pi$ ). This asymmetrical feature is caused by the modulation phase in the coupling loop, which corresponds to an effective synthetic magnetism [70–75]. These findings mean that we can switch a multimode quantum device between separable and entangled states by tuning  $\Theta$ .

*Noise-tolerant optomechanical entanglement*—The DMB entanglement provides a feasible way to create and protect fragile quantum resources against dark modes, and can enable the construction of noise-tolerant quantum devices. We can see from Fig. 3(a) that, in the DMU regime, quantum entanglement emerges only when  $\bar{n} \ll 1$  (see solid curves), while in the DMB regime, it can persist for a threshold value near  $\bar{n} \approx 10^3$  (see dashed curves), which is *three orders of magnitude* larger than that in the DMU regime. In Fig. 3(b), we plot  $E_{N,j}$  versus the cavity-field decay rate  $\kappa$  in both the DMB and DMU regimes. In the DMU regime, quantum entanglement is fully destroyed ( $E_{N,j} = 0$ ) by the thermal noise in the dark mode, and it is independent of  $\kappa$  (see the lower horizontal solid lines). However, in the DMB regime, strong entanglement is generated owing to dark-mode breaking, and  $E_{N,j}$  exist only in the resolved-sideband regime  $\kappa/\omega_m < 1$  (see the upper dashed curves). The maximal entanglement is located at  $\kappa \approx 0.2\omega_m$ , which is consistent with the typical deep-resolved-sideband conditions [25–28, 33].

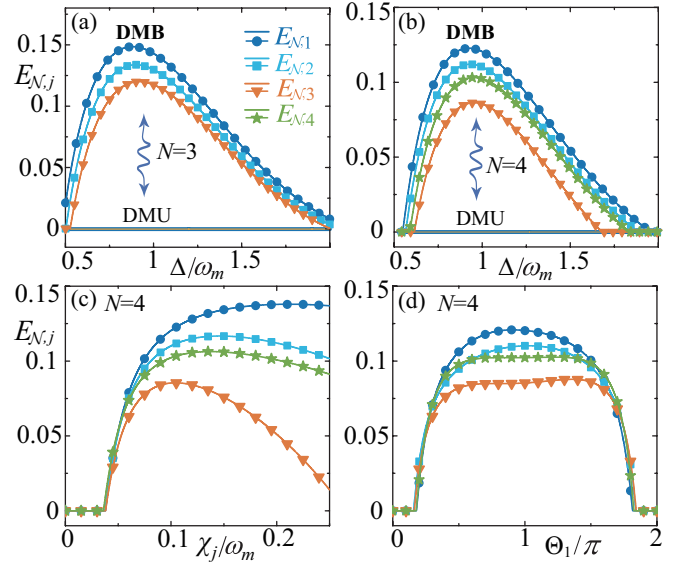


FIG. 4. (a,b)  $E_{N,j}$  versus  $\Delta$  in the DMU ( $\chi_j = 0$ , horizontal solid lines) and DMB ( $\chi_j/\omega_m = 0.1$  and  $\Theta_1 = \pi$ ,  $\Theta_{j \in [2, N-1]} = 0$ , marked by symbols) regimes, for (a)  $N = 3$  and (b)  $N = 4$ . (c)  $E_{N,j}$  versus  $\chi_j$  when  $\Theta_1 = \pi$ . (d)  $E_{N,j}$  versus  $\Theta_1$  when  $\chi_j/\omega_m = 0.1$ . Here  $\bar{n}_j = 10$  and other parameters are the same as those in Fig. 2.

*Entangled optomechanical networks.*—We generalize the DMB approach for optomechanical networks where an optical mode couples to  $N \geq 3$  vibrational modes via the optomechanical interactions  $H_{\text{opc}} = \sum_{j=1}^N g_j c^\dagger c (d_j + d_j^\dagger)$ , and the nearest-neighbor vibrational modes are coupled through the phase-dependent phonon-exchange couplings  $H_{\text{pec}} = \sum_{j=1}^{N-1} \chi_j (e^{i\Theta_j} d_j^\dagger d_{j+1} + \text{H.c.})$  [see Fig. 1(b)]. We have confirmed that these phases are governed by the term  $\sum_{\nu=1}^{j-1} \Theta_\nu$  ( $j \in [2, N]$ ) [81], and hence we assume  $\Theta_1 = \pi$  and  $\Theta_{j \in [2, N-1]} = 0$  in our simulations.

We demonstrate that when turning off synthetic magnetism (i.e.,  $\chi_j = 0$ ), there exists only a single bright mode  $\mathcal{B} = \sum_{j=1}^N \delta d_j / \sqrt{N}$  and  $(N - 1)$  dark modes, with the  $l$ th dark mode expressed as  $\mathcal{D}_{l \in [1, N-1]} = \sum_{j=1}^N \delta d_j e^{2\pi i(j - \frac{N+1}{2})l/N} / \sqrt{N}$ . In the presence of synthetic magnetism (i.e.,  $\chi_j \neq 0$ ), all the dark modes are broken by tuning  $\Theta_1 \neq 2n\pi$  for an integer  $n$  [81]. This provides a possibility of *switching* between the DMB and DMU regimes in optomechanical networks.

We reveal that light and all the vibrational modes are separable ( $E_{N,j} = 0$ , see the lower horizontal solid lines) in the DMU regime, but they are strongly entangled ( $E_{N,j} > 0$ , see the upper symbols) in the DMB regime [see Figs. 4(a) and 4(b)]. Larger entanglement for optomechanical networks can be achieved for the red-sideband resonance (i.e.,  $\Delta \approx \omega_m$ ) and  $\chi_j/\omega_m \in [0.1, 0.15]$  when  $\Theta_1 = \pi$  [see Figs. 4(a), 4(b), and 4(c)]. Physically, the resulting synthetic gauge fields lead to breaking all the dark modes, and make the light-vibration

entanglement networks feasible [see Fig. 4(d)]. This indicates that the entanglement networks, with immunity against dark modes, can be realized by applying the DMB mechanism to optomechanical networks. These findings provide a way to *protect large-scale fragile quantum resources from thermal noises and dark modes*.

*Conclusion.*—We showed how to achieve both dark-mode-immune and noise-tolerant optomechanical entanglement via synthetic magnetism. This is realized by introducing a loop-coupling configuration with a combination of optomechanical interactions and phase-dependent phonon-hopping couplings. We revealed that both bipartite and genuine tripartite light-vibration entanglement arise from the DMB mechanism, without which they vanish. In particular, we found that the threshold thermal phonon number for preserving the DMB entanglement can be up to three orders of magnitude of that in the DMU regime. This study could enable constructing large-scale entanglement networks with dark-mode immunity and noise tolerance.

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