Nonreciprocal Topological Phonon Transfer Independent of Both Device Mass and Exceptional-Point Encircling Direction

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Imposing topological operations encircling an exceptional point (EP) engenders unconventional one-way topological phonon transfer (TPT), strictly depending on the direction of EP-inclusive control loops and inherently limited to the small-mass regime of practical resonators. We here show how to beat these limitations and predict a mass-free unidirectional TPT by combining topological operations with the Fizeau light-dragging effect, which splits countercirculating optical modes. An efficient TPT happens when light enters from one chosen side of the fiber but not from the other, leading to a unique nonreciprocal TPT, independent of the direction of winding around the EP. Unlike previous proposals naturally sensitive to both mass and quality of quantum devices, our approach is almost immune to these factors. Remarkably, its threshold duration of adiabatic control loops for maintaining an optimal TPT can be easily shortened, yielding a top-speed-tunable perfect TPT that has no counterpart in previous demonstrations. The study paves a quite-general route to exploiting profoundly different chiral topological effects, independent of both EP-encircling direction and device mass.

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Nontrivial topologies, strongly affected by the presence of non-Hermitian degeneracies [1–18], give rise to fascinating and counterintuitive unidirectional topological phenomena in slowly evolving non-Hermitian systems when an exceptional point (EP) is encircled, such as one-way topological phonon transfer (TPT) or mode conversion [19–29], chiral phase accumulation [30,31], and nonadiabatic jumps [32,33]. Nevertheless, these topological behaviors are intrinsically limited by the strict dependence on the direction of adiabatic EP-including encirclements in parameter space [19,20], resulting in a strong suppression effect on both topological response and its nonreciprocity. Specifically, for an initial excitation in a chosen mode, an efficient TPT is observed when executing an adiabatic trajectory enclosing clockwise an EP; while for counterclockwise encircling, no TPT happens irrespective of the tuning of system parameters [19,20]. Such advances with an EP-encircling-direction-dependent limitation have been used in topological manipulation and switching of light, sound, and microwaves [20,32–36].

Recent theoretical and experimental progresses have shown that topology can provide robustness in quantum resources against environmental perturbations and random fabrication imperfections, e.g., impurities and disorders [37–46]. The practical applicability for modern chiral topological technologies, however, has challenged such advances by demonstrating that the presently established topological behaviors are naturally limited to the small-mass regime (femto- or nanogram) of practical resonators [19–33]. This is because the decrease in the zero-point motion of the resonator with increasing its mass results in a greatly reduced coupling strength, making all the topological responses hard to achieve. In terms of these long-standing limitations, where the unidirectional topological physics is not only strictly dependent on the EP-encirclement direction but also can be easily destroyed in the large-mass regime, the exploitation of a new nonreciprocal topology independent of the winding direction of the EP, as well as shielding the topological behavior from the mass disturbances in practical setups, is highly desirable.

We here propose to induce a versatile yet unique one-way TPT completely independent of the EP-encircling direction, and reveal its counterintuitive insensitiveness to device masses. This happens because of the synergy of the topological operations and Fizeau drag [47–57], leading to the difference of the refractive index that light experiences. We find that injecting light from one chosen side of the fiber results in a TPT, whereas injecting it from the other side does not. This produces a fundamentally different topological nonreciprocity independent of the encirclement direction of the adiabatic loop enclosing the EP, which is otherwise unattainable in conventional schemes [19–33].

As opposed to previous studies, where the topological responses are generally deteriorated or even completely destroyed with increasing (decreasing) the mass (quality) of practical setups [19–33], our proposal is almost immune to

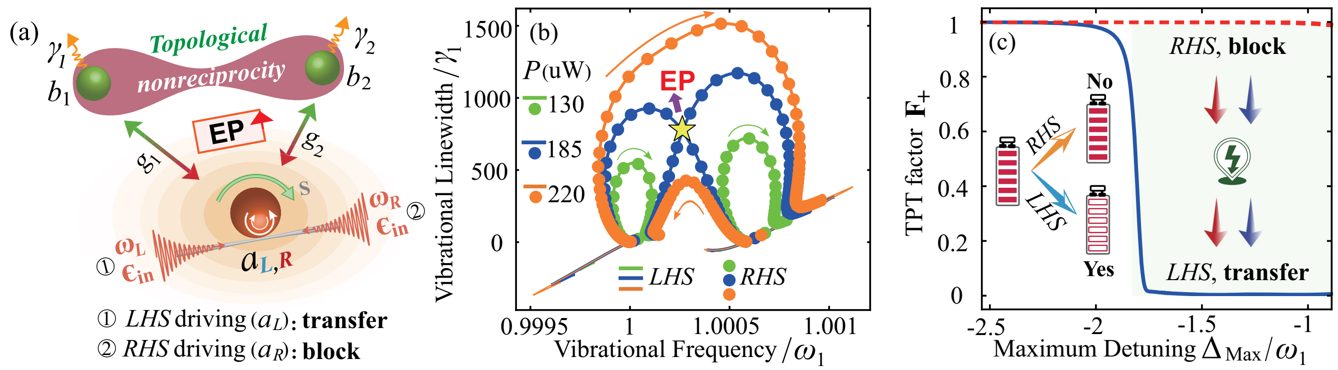

FIG. 1. (a) Schematic of the device consisting of a clockwise spinning silica-microsphere cavity $a_{R,L}$ (with angular velocity S) coupled to two vibrations b_j (with decay rates γ_j) through light-motion interactions g_j. The spinning microsphere is driven by a laser from the left-hand side (lhs) or right-hand side (rhs) of the fiber, yielding an anticlockwise (a_L) or a clockwise (a_R) optical mode, respectively. (b) Damping rate and resonance frequency of vibrational normal modes versus the driving detuning $\Delta \in [-2\omega_1, 0]$ and laser power P, when the laser enters from the lhs or rhs, under $S = 400$ Hz. (c) $\mathbf{F}_+$ versus the maximal driving detuning Δ_{Max}, when light is injected from the lhs or rhs.

these detriments without the need of using any high-cost low-loss materials and noise filters at the expense of the system complexity [58,59]. Its underlying physics is analogous to the Doppler effect extensively demonstrated in various nontopological areas [60,61], and this mechanism leads to a complete compensation for the mass- and loss-induced detrimental reflections, owing to a drastic enhancement in the intracavity relative photon number. In a broader view, our study sheds new light on bridging the topological operations and the Fizeau light-dragging effect, and offers exciting opportunities of revealing new one-way topological behavior, with both independence on the EP-encircling direction and immunity against the device mass.

Model and its EP.—We consider a multimode optomechanical configuration, where a spinning optical-microsphere cavity coupled to two motional modes [36,62–72] is positioned close to a tapered region of a single-mode telecommunication fiber [Fig. 1(a)]. The evanescent coupling of light into this spinning setup is caused by the tapered region of the fiber, and via the same coupler, light can be coupled out through the other side of the fiber [47]. Consequently, both input and output ports can be simultaneously accessed by each side of the fiber, and dependent on the input port, light circulates in the microsphere in either clockwise or counterclockwise direction, resulting in an optical mode a_R or a_L (with resonance frequencies $\omega_c^{R,L}$), respectively. The spinning-system Hamiltonian reads ($\hbar = 1$)

$$\mathcal{H} = \omega_c^l a_l^\dagger a_l + \sum_{j=1,2} [\omega_j b_j^\dagger b_j + g_j a_l^\dagger a_l (b_j^\dagger + b_j)] + i\sqrt{\kappa_{\mathrm{in}}}\epsilon_{\mathrm{in}}(a_l^\dagger e^{-i\omega_l t} - \mathrm{H.c.}), \quad \text{for } l = R, L, \tag{1}$$

where $b_j^\dagger$ (b_j) is the creation (annihilation) operator of the jth vibrational mode with resonance frequencies ω_j. The g_j terms denote optomechanical interactions with m_j being the resonator mass, and the $\epsilon_{\mathrm{in}} = \sqrt{P/(\hbar\omega_l)}$ term depicts the system driving, with ω_l (P) being the frequency (power) of the laser, and κ (κ_{in}) denoting the linewidth (input-coupling rate) of the optical field. Experimentally, the aerodynamic process plays a vital role in fiber-resonator interactions for the rotating setup, which drags air into the region between the sphere and tapered fiber, yielding an air-lubrication layer in this region. The thin air film then lets the fiber fly at a few nanometers above the sphere [73]. Owing to the "self-adjustment" process, the taper floats back to its initial position, once it is caused to rise higher than the stable-equilibrium height by any perturbation. The critical coupling of light into the microsphere is enabled via the self-adjustment effect, whereby an optical drag identical in size but opposite in sign is experienced by the countercirculating light [73].

To explore how the Fizeau drag of light causes chirality, the relativistic addition of velocities is taken into account when the periphery of the spinning sphere is moving away from the output or towards the input ports. In light of these considerations, optical paths of counterpropagating light beams are different attributed to the rotation, leading to the irreversible refractive indices for $a_{R,L}$,

$$\zeta_{R,L} = \zeta[1 \pm S\zeta r(\zeta^{-2} - 1)/c], \tag{2}$$

where ζ, S, r, and c are the refractive indices of materials, the spinning angular velocity of the microsphere, the sphere radius, and the light speed in vacuum, respectively. Evidently, a Sagnac-Fizeau shift is experienced by the light mode, i.e., $\omega_c \to \omega_c + \delta_s$ where $\delta_s = \pm S\Lambda$ with $\Lambda = \zeta r \omega_c[1 - 1/\zeta^2 - (\lambda/\zeta)(d\zeta/d\lambda)]/c$ for the nonspinning optical frequency ω_c and the light wavelength λ. The relativistic origin of the Sagnac effect is characterized by the dispersion term $d\zeta/d\lambda$ [47]. By clockwise spinning the microsphere, the resulting positive (negative) value of δ_s corresponds to a_R (a_L) with $\omega_c^{R,L} = \omega_c \pm |\delta_s|$. Here $S = 0$ depicts the standard (i.e., nonspinning) case.

By adiabatically eliminating the optical mode, an effective Hamiltonian for the two motional modes is obtained [73]:

$$\mathcal{H}_{\text{eff}}^{R,L} = \begin{pmatrix} \omega_1 - \frac{i\gamma_1}{2} - ig_1^2\sigma_{R,L} & -ig_1g_2\sigma_{R,L} \\ -ig_1g_2\sigma_{R,L} & \omega_2 - \frac{i\gamma_2}{2} - ig_2^2\sigma_{R,L} \end{pmatrix}, \tag{3}$$

where the complex vibrational susceptibilities $\sigma_{R,L}$, induced by the laser driving, are defined as

$$\sigma_{R,L} = \frac{P\kappa_{\text{in}}[\chi_{R,L}(\omega_0) - \chi_{R,L}^*(-\omega_0)]}{\hbar\omega_l[(\kappa/2)^2 + \Delta_{R,L}^2]}, \tag{4}$$

with optical susceptibilities $\chi_{R,L}(\omega_0) = [\kappa/2 - i(\omega_0 + \Delta_{R,L})]^{-1}$ for $\omega_0 = (\omega_1 + \omega_2)/2$, and $\Delta_{R,L} = \Delta \mp |\delta_s|$ for the driving detuning $\Delta = \omega_l - \omega_c$. An EP can be easily reached by tuning $\sigma_{R,L}$, which needs to control over both $\text{Re}(\sigma_{R,L})$ and $\text{Im}(\sigma_{R,L})$. The real and imaginary parts of the corresponding complex eigenvalues are, respectively, the resonance frequencies and spectral linewidths. To reach and encircle the EP, it is enough to tune P and Δ, which are easily controlled *in situ* with a timing accuracy, high-precision degree, and dynamic range.

To demonstrate the dependence of the EP on the Fizeau light-dragging effect, we show the mechanical spectra versus Δ and P in both lhs- (solid curves) and rhs-driving (symbols) cases [Fig. 1(b)]. For a lower power, each eigenvalue follows an enclosed trajectory, which begins and ends at the same point; while for a higher power, the eigenvalues follow open paths, each of which starts at the ending point of the other. In both cases, the EP (yellow star), where the eigenstates coalesce, emerges by adjusting P and Δ. Counterintuitively, the size of the trajectories in the lhs driving is larger than that in the rhs case, owing to a profoundly different influence from the Fizeau-drag shift of light [47]. We now define a TPT quality factor to quantify the energy transfer from the vibrational normal modes b_+ to b_- [19,73]:

$$\mathbf{F}_+ = |b_+(\tau)|^2/[|b_+(\tau)|^2 + |b_-(\tau)|^2], \tag{5}$$

denoting the fraction of the remaining energy in the b_+ mode after performing control loops, with $|b_\pm(\tau)|$ being the amplitudes of the motion of the normal modes at the end of the loop [73]. Note that before executing control loops, the definition of $\mathbf{F}_+$ needs to satisfy the property that all the energy remains in b_+ (i.e., for the b_+ initialization). Equation (5) clearly shows that $\mathbf{F}_+ \to 1$ implies no TPT from b_+ to b_-; while $\mathbf{F}_+ \to 0$ means an excellent TPT. The definition $\mathbf{F}_- = 1 - \mathbf{F}_+$ describes the fraction of the energy in b_-. In our simulations, to ensure the system stability, we choose experimentally feasible parameters [47,103]: $n = 1.486$, $\lambda = 1064$ nm, $\omega_2/\omega_1 = 1.000\,59$, $r = 0.755$, $g_{1(2)}/\omega_1 = 1.31(1.45) \times 10^{-6}$, $\kappa(\kappa_{\text{in}})/\omega_1 = 0.225(0.089)$, and $\gamma_{1(2)}/\omega_1 = 0.76(1.78) \times 10^{-6}$.

Excellent TPT nonreciprocity independent of the EP-encircling direction.—The described one-way TPT occurs regardless of the EP-encircling direction. For the convenience of understanding its underlying physics, we first consider the case of adiabatically winding around the singularity counterclockwise. Light injected from one chosen side of the fiber experiences a Sagnac-Fizeau shift, whereas light entering from the other side yields an opposite shift. Specifically, we display in Fig. 1(c) $\mathbf{F}_+$ versus Δ_{Max}, in both lhs- and rhs-driving cases. We find that by injecting light from the rhs of the fiber, no TPT is observed ($\mathbf{F}_+ \to 1$, red dashed curves); while by injecting it from the lhs, an excellent TPT occurs ($\mathbf{F}_+ \to 0$, blue solid curves). The resulting nonreciprocal TPT is fully independent of the EP-encircling direction [73], and it has no correspondence to the previously established demonstrations [19–33], where the dependence on the EP-encircling direction is an essential requirement for standard unidirectional topological behaviors.

Mass-insensitive TPT.—It is well known that the topological responses are naturally restricted to the small-mass regime of quantum devices. Our approach, however, paves a feasible route to immunizing topological resources against device masses, and enables the construction of mass-tolerant phononic devices. We show in Fig. 2(a) that for conventional schemes ($S = 0$), no TPT ($\mathbf{F}_+ \to 1$) occurs when the mass ratio $\rho > 10$; in stark contrast to this, by introducing the Fizeau light-dragging effect ($S \neq 0$), an optimal TPT can be achieved regardless of resonator mass ($\mathbf{F}_+ \to 0$, yellow areas). Physically, the TPT is suppressed due to the decrease in the light-vibration coupling with increasing resonator mass, while it can be considerably compensated or enriched because of the Fizeau drag. This indicates that, in general, by simply employing the Fizeau drag of light, the TPT can be almost independent of the device mass. A sharp variation

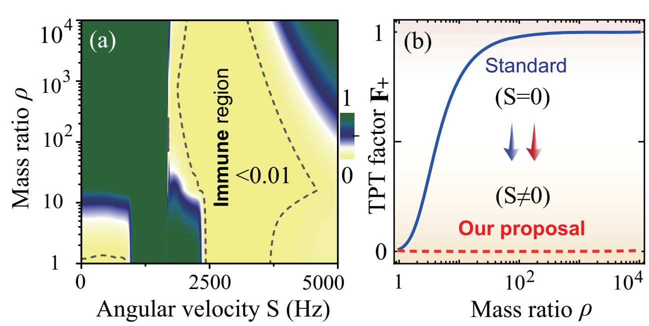

FIG. 2. (a) $\mathbf{F}_+$ versus the mass ratio $\rho = \rho_1$ and S in the lhs-driving case. (b) $\mathbf{F}_+$ versus ρ for standard ($S = 0$) and our approaches ($S = 3$ kHz).

in Fig. 2(a) is resulted from the deactivation of the self-adjustment process [73].

To further show this counterintuitive immunity against the mass, we compare the standard and our proposals [see Fig. 2(b)]. In conventional schemes, the TPT sharply deteriorates with increasing ρ ($\mathbf{F}_+ \to 1$, blue curves); while our method allows us to reach the mass-immune TPT ($\mathbf{F}_+ = 0$, red curves). In view of elucidating and verifying its underlying physics, we define effective light-motion couplings $G_j^{R,L}$ and mean photon numbers $N_{R,L}$ by

$$G_j^{R,L} = g_j\sqrt{N_{R,L}/\rho_j}, \tag{6a}$$

$$N_{R,L} = \frac{P\kappa_{\rm in}}{[(\Delta \mp |\delta_s|)^2 + (\kappa/2)^2]\omega_l\hbar}, \tag{6b}$$

where $\rho_j = m_j/m_0$. It clearly shows that $G_j^{R,L}$ significantly decreases with increasing ρ_j; however, the application of Fizeau drag to topological operations results in an enhancement of $N_{R,L}$, which giantly compensates or even amplifies the effective optomechanical interactions. This physical mechanism is analogous to the Doppler effect extensively used in various areas of nontopological physics [60,61]. These findings demonstrate that the mass-induced detrimental reflections can be significantly suppressed by introducing the Fizeau light-dragging effect and as a result, one can achieve a nearly ideal TPT. To realize an excellent TPT, we predict a threshold angular speed $S_{\rm thr}$ corresponding to ρ:

$$S_{\rm thr} = \left\{-\Delta \pm \sqrt{[4\Delta^2 - (\rho - 1)\kappa^2]/(4\rho)}\right\}/\Lambda. \tag{7}$$

Tolerance for setup-quality factors.—More importantly, the resulting nonreciprocal TPT provides a novel way to enable the practical bad-quality setups to become ideal, and it is beneficial to achieving quality-immune topological phononics. Concretely, by employing the Fizeau drag of light, the TPT beyond the limitations of quality factors reaches around $S = 400$ Hz [Figs. 3(a) and 3(b)]. To render a more complete portrait of this counterintuitive behavior, we show in Figs. 3(c) and 3(d) how the TPT changes in both the high- and low-quality regimes. In the low-quality regime, the TPT almost does not occur ($\mathbf{F}_+ \to 1$) for the standard scheme, but it becomes feasible ($\mathbf{F}_+ \to 0$) for our spinning method. Especially, its TPT performance can be improved up to 3 (2.5) orders of magnitude compared with the conventional scheme [19]. These results indicate that applying the Fizeau drag of light to topological operations establishes not only a giant enhancement in the phononic isolation, but also offers the possibility of immunizing the topological behavior against the disturbances of device quality factors.

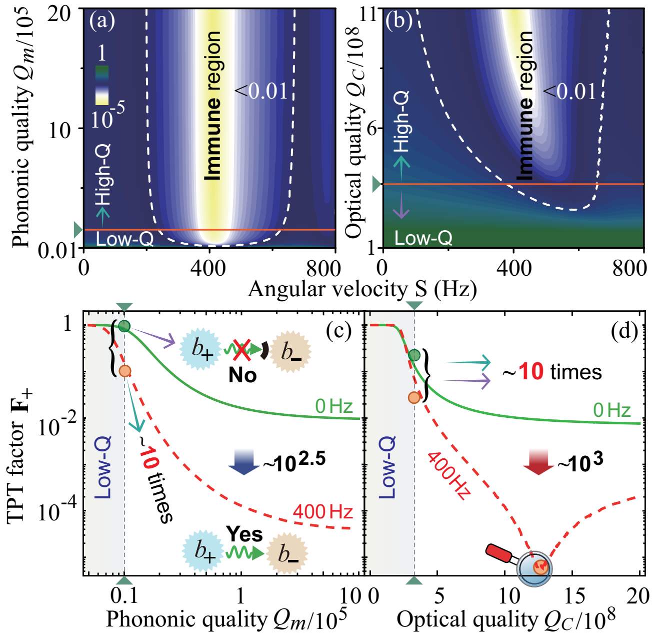

FIG. 3. (a) $\mathbf{F}_+$ versus the phononic quality factor $Q_m = Q_2$ and S in the lhs-driving case. (b) $\mathbf{F}_+$ versus the optical quality factor Q_c and S. $\mathbf{F}_+$ versus (c) Q_m and (d) Q_c when $S = 0$ Hz and 400 Hz.

Controllability of top speed of the perfect TPT.—In Fig. 4 and Eq. (8), by rapidly encircling the EP ($\tau \to 0$), no TPT happens ($\mathbf{F}_+ \to 1$, $\mathbf{F}_- \to 0$); while with adiabatically winding around this EP ($\tau \gg 1$ ms [19]), an excellent TPT is observed ($\mathbf{F}_+ \to 0$, $\mathbf{F}_- \to 1$). Remarkably, in the lhs-injecting case, the threshold duration $\tau_{\rm thr}$ of the adiabatic control loops for preserving a perfect TPT is nearly 5 times less than that in the standard schemes. Note that the threshold duration $\tau_{\rm thr}$ corresponds to a top speed of the perfect TPT. To clearly demonstrate these counterintuitive phenomena, we show the relationship between $\mathbf{F}_+$ and $\tau_{\rm thr}$:

$$\begin{aligned} 0 < &\mathbf{F}_+(\tau < \tau_{\rm thr}) \leqslant 1\text{: No or partial TPT;} \\ &\mathbf{F}_+(\tau \geqslant \tau_{\rm thr}) = 0\text{: Optimal TPT.} \end{aligned} \tag{8}$$

For the lhs driving, a leftward shift appears compared to the standard case, leading to speed up of the TPT. This paves a route to a top-speed-tunable excellent TPT.

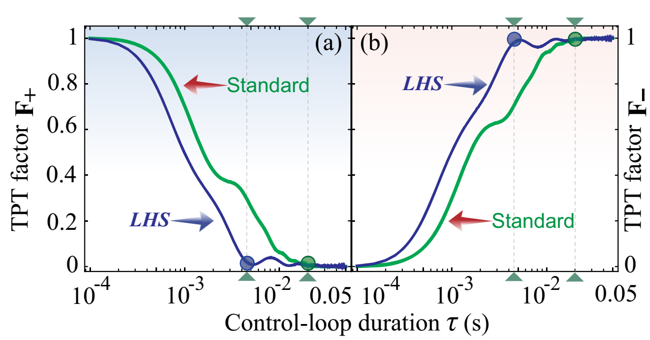

FIG. 4. (a) $\mathbf{F}_+$ and (b) $\mathbf{F}_-$ versus the control-loop duration τ in the standard and lhs-driving cases when $S = 800$ Hz.

Discussions, conclusions, and outlook.—To extensively enlarge the discussions on their experimental feasibility, we have presented detailed analyses on both simulation and experiment parameters using realistic state-of-the-art experimental conditions [73]. An excellent agreement between proposed and simulated results demonstrates that our main conclusions and potential applications are relevant for the realistic state-of-the-art experiments. Building upon widely recognized experimental demonstrations focusing on the semiclassical EP [19,74,104–106], the cryogenic optomechanical platforms can be employed to minimize the impact of temperature. In these mature experiments, even though considering the semiclassical EP, experimental results are in complete agreement with theoretical predictions. Note that our methods dealing with the temperature effect on the TPT are entirely based on these well-established experimental works, and the temperature effect on the nonreciprocal TPT can also be considered by studying the quantum EPs of non-Hermitian Hamiltonians and Liouvillians [75]. In an earlier investigation [20], asymmetric mode transfer was distinctly visualized on the Riemann surfaces. In principle, our counterintuitive findings can also be characterized through the Riemann surfaces. However, our endeavors have revealed that because of the spinning of our microsphere cavity, this approach remains challenging for readers to grasp effectively. Consequently, we embrace the Fizeau light-dragging effect [47] as a more accessible and coherent framework for interpretation, which is comprehensible to a broader audience.

In conclusion, we showed a both mass-insensitive and EP-encircling-direction-independent nonreciprocal TPT arising from the Fizeau light-dragging effect, without which it vanishes. Our work on the TPT differs from what is known in optomechanical lattices or crystals with energy bands, mainly because we are not focused on a 1D chain (2D honeycomb lattice) [76], or multiscale optomechanical crystals [21], but on three-mode optomechanics without energy bands [19]. Our study describes a general mechanism, and maps a new way of manipulating one-way TPT, independent of the encircling direction of the EP. In a broader view, it enables constructing novel topological chiral phononics with both device-mass tolerance and TPT-velocity tunablility. Our approach can be extended to a more general non-Hermitian N-state system [73], holding significant promise for exploring higher-order-EPs and non-Abelian braiding properties [77–79]. It can open avenues for further exploration in the intersection of the Fizeau light-dragging effect, non-Hermitian N-state systems, and non-Abelian nature, and contribute novel insights to unique one-way topology.

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