

Nonreciprocal quantum synchronization

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Nonreciprocal physics is garnering enormous attention in both classical and quantum resource fields. Surprisingly, previous demonstrations have not explored nonreciprocal quantum synchronization of phonons, one of the most obvious examples of nonreciprocal quantum resources. Here we fill this gap to demonstrate the possibility of nonreciprocal quantum synchronization, revealing its counterintuitive robustness against random fabrication imperfections and thermal noise of practical devices. Specifically, phonons are synchronized in a chosen direction of light (magnetic field) but unsynchronized in the other, yielding a unique nonreciprocity of quantum synchronization. This happens by harnessing the synergy of the Sagnac and magnon-Kerr effects, leading to an opposite Sagnac-Fizeau shift and an exceptional magnon-Kerr-induced transition. Unlike previous proposals naturally restricted to the low-imperfection regime, our approach beats this limitation, owing to the magnon-Kerr-induced improvement in the resonator resilience. The study lays the foundation for generating fragile-to-robust nonreciprocal quantum resources.

Nonreciprocal components, enabling routing light signals along one desired path but strongly attenuating them along the other, have been found a variety of unique applications ranging from interfering-insensitive signal processing, noise-free radio and radar, to invisible cloaking or sensing¹. To suppress undesirable interferences, spurious modes, and unwanted signal paths², nonreciprocal optical and acoustic configurations have been implemented^{3–17}, by exploiting magneto-optical or ferrite materials¹⁸, optical nonlinearities^{19–21}, chiral atomic states²², and temporal modulation^{23–27}. Yet previously established demonstrations are mainly interested in nonreciprocal engineering in the classical domain, but less in the quantum regime^{1–27}. By contrast, nonreciprocal quantum control is expected to enable the exploitation of nontrivial quantum structures and to actively stimulate explorations of new nonreciprocal quantum resources²⁸.

Recently, partial proposals have begun focusing on nonreciprocal quantum engineering, that are devoted to designing a new generation of thermal rectifiers or diodes²⁹ and one-way quantum processors^{30–41}, by employing nonreciprocal interactions^{29–31}, the Sagnac effect^{32–37}, or the nonlinearity^{38–41}. Practical applicability of modern nonreciprocal quantum technologies, however, has challenged such progress by demonstrating that inherent random fabrication imperfections (e.g.,

large mass and/or high loss) and detrimental noise of practical devices profoundly suppress or even completely destroy all related quantum resources^{28–41}. In view of the significance of these outstanding challenges, the exploitation of a profoundly different nonreciprocal control of quantum resources, as well as shielding them from both imperfection and noise disturbances in realistic setups, is ultimately required.

Here we describe how to construct an extraordinary nonreciprocal engineering of quantum synchronization, a crucial quantum resource of modern quantum technologies⁴², and demonstrate its counterintuitive robustness against both random fabrication imperfections and thermal noise. Physically, quantum synchronization extends classical synchronization into the quantum regime, revealing nontrivial correlations in quantum dynamics of coupled oscillators, spins, or fields⁴³. Unlike its classical counterpart, quantum synchronization manifests through correlations in quantum observables and is constrained by quantum noise and noncommutativity, offering a route to controlling collective quantum dynamics.

Nonreciprocal quantum synchronization occurs via the synergy of the Sagnac effect splitting countercirculating optical modes^{32,33} and the magnon-Kerr effect stemming from magnetocrystalline anisotropy^{44–49}. Specifically, injecting light (magnetic field) from one

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chosen orientation results in quantum synchronization, whereas applying it along the other does not, giving rise to a unique non-reciprocal quantum synchronization. Very recently, nonreciprocal synchronization of active quantum spins has been demonstrated through engineered nonreciprocal coupling⁵⁰. Unlike previous schemes, where quantum resources are generally deteriorated or even fully destroyed with increasing mass, loss, and/or noise^{28–51}, our approach is robust to these detrimental factors, without the need of utilizing any high-cost low-loss materials and noise filters at the expense of system's complexity^{52,53} or any topological structures^{54–60}. This is attributed to harnessing the power of the introduced magnon-Kerr nonlinearity, which dramatically improves the resilience of resonators. Physically, the effect of the tailored magnon-Kerr nonlinearity is twofold. It introduces an exceptional magnon-Kerr-induced transition and a significant modulation in effective coupling strengths, which are responsible for nonreciprocal quantum synchronization and the robustness against decoherence, respectively.

Results

Physical model and its Hamiltonian

We consider a hybrid quantum platform consisting of a silica microsphere in combination with a yttrium iron garnet (YIG) sphere, coherently coupled to each other via direct physical contact⁶¹. The described system has already been implemented with state-of-the-art technology to enable new architectures for coherent coupling between magnons, phonons, and photons⁶¹. Concretely, uniform magnons are supported by the YIG sphere using an external magnetic field H , and the magnetostrictive force, exciting phonons by a microwave, leads to the coupling of magnons to phonons. In the silica microsphere, phonons are coupled to photons via radiation pressure and the photoelastic effect^{62–72}. The phonon-phonon interaction χ stems from direct touch of a spinning silica microsphere with a counter-rotating YIG sphere, both locked at the same angular speed Ω . The system Hamiltonian reads ($\hbar = 1$)

$$\begin{aligned} \mathcal{H} = & \sum_{o=a,m} \Delta_o o^\dagger o + \sum_{j=1,2} \omega_j b_j^\dagger b_j - \chi (b_1^\dagger b_2 + b_2^\dagger b_1) \\ & - g_m m^\dagger m (b_1^\dagger + b_1) - g_a a^\dagger a (b_2^\dagger + b_2) \\ & + K (m^\dagger m)^2 + i \sum_o (\xi_o o^\dagger - \xi_o^* o), \end{aligned} \quad (1)$$

where a , m , and b_j are the annihilation operators of photons, magnons, and phonons, with resonance frequencies (decay rates): ω_a (κ_a), ω_m (κ_m), and ω_j (γ_j), respectively. The phonon-, magnon-, and photon-phonon interactions are, respectively, denoted by the χ , g_m , and g_a terms. The coupling rate between a single excitation and the j th mechanical mode scales as $g_j = \eta x_{\text{ZPM},j}$ (with $g_{1(2)} = g_{m(a)}$), where η_j quantifies the coupling strength to the resonator's position $x_j(t)$, and $x_{\text{ZPM},j} \sim \sqrt{\hbar/(2m_j\omega_j)}$ is the zero-point motion, with m_j being the microsphere-resonator mass. For a large-mass resonator, a greatly reduced coupling strength g_j results from the decrease in $x_{\text{ZPM},j}$ with increasing m_j , and it makes quantum synchronization hard to achieve. The K and ξ_o terms respectively describe the magnon-Kerr nonlinearity and the system drivings with frequency $\omega_{d,o}$ and detuning $\Delta_o = \omega_o - \omega_{d,o}$.

Sagnac-Fizeau and magnon-Kerr transitions

We expand the operators $o \in \{a, m, b_j\}$ as sums of their classical averages $\langle o \rangle$ and quantum fluctuations δo . By defining the quadratures $\delta X_o = (\delta o^\dagger + \delta o)/\sqrt{2}$ and $\delta Y_o = i(\delta o^\dagger - \delta o)/\sqrt{2}$, and the corresponding Hermitian input-noise operators $X_o^{\text{in}} = (o_{\text{in}}^\dagger + o_{\text{in}})/\sqrt{2}$ and $Y_o^{\text{in}} = i(o_{\text{in}}^\dagger - o_{\text{in}})/\sqrt{2}$, we obtain linearized quantum Langevin equations $\mathbf{u}^*(t) = \mathbf{A}\mathbf{u}(t) + \mathbf{N}(t)$, with the fluctuation operator vector $\mathbf{u}(t)$, noise operator vector $\mathbf{N}(t)$, and coefficient matrix $\mathbf{A}$ (see Supplementary Information). System properties are inferred utilizing the covariance

matrix $\mathbf{V}$ with $\mathbf{V}_{kl} = [\langle \mathbf{u}_k \mathbf{u}_l^\dagger \rangle + \langle \mathbf{u}_l \mathbf{u}_k^\dagger \rangle]/2$ ⁷³, fulfilling the Lyapunov equation $\mathbf{V}^* = \mathbf{A}\mathbf{V} + \mathbf{V}\mathbf{A}^\dagger + \mathbf{Q}$.

The Sagnac effect arises in rotating reference frames, where counterpropagating waves traveling along a closed loop accumulate a relative phase shift proportional to the rotation rate^{32,33}. This relativistic interference phenomenon underpins modern gyroscopes and enables directional sensitivity in photonic and phononic systems. In stark contrast, the magnon-Kerr effect refers to a nonlinear frequency shift of magnon modes induced by magnon-magnon interactions in a magnetically ordered material^{44–49}. This self-phase modulation leads to intensity-dependent magnon dynamics, analogous to the optical-Kerr effect, enabling tunable nonlinearity in hybrid quantum systems. Arising from magnetocrystalline anisotropy, this nonlinear interaction gives rise to tunable magnon dynamics and facilitates nonperturbative phenomena such as bistability and nonreciprocal signal propagation^{44–49}. To show how the Sagnac-Fizeau (magnon-Kerr) effect enables nonreciprocal quantum synchronization, we take the relativistic addition into account when light (magnetic field) enters from the right ($\mathcal{CD}$) or left ($\mathcal{OD}$). Specifically, the rotation of the silica microsphere causes the optical paths of counterpropagating light beams within the cavity to diverge, resulting in an irreversible refractive index for a clockwise or counterclockwise optical mode $\zeta \rightarrow \zeta [1 \pm \Omega \zeta r (\zeta^2 - 1)/c]$, with ζ , Ω , c and r being the refractive indices of materials, spinning angular velocity, light speed in vacuum, and silica-microsphere radius, respectively. Meanwhile, the magnetocrystalline anisotropy in the YIG sphere leads to the difference of the magnon frequencies. Evidently, an opposite Sagnac-Fizeau shift and a unique Kerr-nonlinearity-induced shift are, respectively, experienced by the resonance frequencies of the counterpropagating light mode and the magnon mode^{32,44,45}.

$$\omega_o \rightarrow \omega_o + \sigma_o, \text{ for } o = a, m, \quad (2)$$

where $\sigma_a = \pm \Omega \Lambda$ and $\sigma_m = 4K N_m$ with $\Lambda = \zeta r \omega_c [1 - 1/\zeta^2 - (\lambda/\zeta)(d\zeta/d\lambda)]/c$ for light wavelength λ and mean magnon numbers N_m . Note that the dispersion term $d\zeta/d\lambda$ is small in typical materials (up to ~1%) and characterizes the relativistic origin of the Sagnac effect³². The relativistic-origin shifts $\sigma_{a(m)} < 0$ and $\sigma_{a(m)} > 0$ correspond to the cases with the laser (magnetic field) input from the right ($\mathcal{CD}$) and left ($\mathcal{OD}$), respectively [Fig. 1(c)]. Nonreciprocal quantum synchronization happens because of an opposite frequency shift for counterpropagating optical modes or a peerless Kerr-nonlinearity-induced transition for magnon modes.

Quantum synchronization measure

For two continuous-variable systems described by dimensionless canonical variables $O_b(t)$ for ($O = X, Y$) representing the evolution of two subsystems in phase space, a complete synchronization is attained when the quantities $O_-(t) := [O_b(t) - O_c(t)]/\sqrt{2}$ approach zero asymptotically for sufficiently large times⁴³. However, extending the aforementioned concepts to quantum mechanical systems poses challenges, as fundamental limits may exist that hinder the exact fulfillment of the conditions described above^{74–76}. By defining dimensionless quantities $O_b(t)$ as the quadrature operators satisfying the canonical commutation rules $[X_{b_j}(t), Y_{b_j}(t)] = i\delta_{jj}$ ^{77,78}, the relative coordinates $O_-(t)$ can represent the generalized position and momentum operators for the same (antisymmetric) mode. Consequently, the uncertainty principle precludes satisfying the precise condition necessary for a complete classical synchronization. To quantify this, we designate $O_-(t)$ as synchronization errors and introduce the following metric:

$$\mathcal{S}_c(t) := \left\langle \sum_o O_-(t)^2 \right\rangle^{-1}, \text{ for } O = X, Y, \quad (3)$$

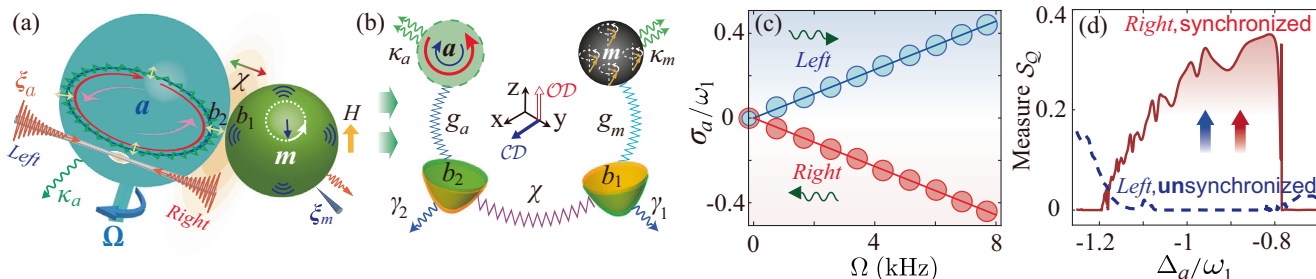


Fig. 1 | Model and Sagnac-effect-induced nonreciprocal quantum synchronization. **a** Schematic of hybrid quantum devices consisting of a clockwise-spinning silica microsphere (with angular velocity Ω) and a YIG sphere, both supporting phonons respectively excited by the radiation-pressure interaction g_a induced by circulating optical fields or by the magnetostrictive coupling g_m induced by microwave-driven magnons. The phonon-phonon coupling χ originates from direct physical contact between a spinning silica microsphere and a counter-rotating YIG sphere, both maintained at a constant angular velocity Ω . The clockwise-spinning silica microsphere is driven by light from the left or right of the fibre, resulting in a counterclockwise or clockwise optical mode, respectively. The externally applied magnetic field is aligned parallel to either one chosen direction (CD , i.e., $[110]$) or the

other direction (OD , i.e., $[100]$) wrt the crystallographic axes of the YIG sphere, leading to a magnon-Kerr coefficient $K < 0$ or $K > 0$, respectively (see Supplementary Information). **b** Schematic of the coupling between magnons, phonons, and photons. **c** Sagnac-Fizeau shift σ_a versus Ω when light enters from the left or right of the fibre. **d** Quantum synchronization measure S_Q versus the optical detuning Δ_a when light is injected from the left or right of the fibre. Here we set $\omega_1 = 2\pi \times 10$ MHz as a reference unit of frequency and choose experimentally feasible parameters^{32,33,43–46,61}: $\omega_2/\omega_1 = 1.005$, $\gamma/\omega_1 = 0.005$, $\kappa_{a(m)}/\omega_1 = 0.15$ (0.2), $g_{a(m)}/\omega_1 = 0.005$, $\chi/\omega_1 = 0.02$, $\Delta_{a(m)}/\omega_1 = -1.005(-1)$, $\bar{n}_j = 0$, $\mathcal{E}_{a(m)}/\omega_1 = 35$, $K = 0$, and $\Omega = 6$ kHz.

quantifying the extent of complete synchronization achieved by the system, where $\langle \dots \rangle$ denotes the expectation value taken with respect to the density matrix of quantum systems. Then, the Heisenberg principle requiring $\langle X_-(t)^2 \rangle \langle Y_-(t)^2 \rangle \geq 1/4$ is observed, therefore

$$S_c(t) \leq \left[\langle X_-(t)^2 \rangle \langle Y_-(t)^2 \rangle \right]^{-1/2} / 2 \leq 1, \quad (4)$$

establishing a universal limit on the complete synchronization achievable between two continuous-variable systems. Conversely, in a purely classical framework, $S_c(t)$ is theoretically unbounded⁴³. A small value of $S_c(t)$ arises from two sources: (i) non-zero mean values of $O_-(t)$, and/or (ii) significant variances in these operators. The former scenario can be seen as a classical systematic error⁴³; whereas the latter stems from the effects of thermal and quantum noise. We can readily eliminate classical systematic errors from the synchronization measurement, shown in Eq. (3), just following a change of variables⁴³:

$$O_-(t) \rightarrow \delta O_-(t) = O_-(t) - \langle O_-(t) \rangle. \quad (5)$$

This gives rise to quantum synchronization measure of phonons:

$$S_Q(t) : = \left\langle \sum_o \delta O_-(t)^2 \right\rangle^{-1}. \quad (6)$$

Quantum synchronization happens when $S_Q > 0$ but not when $S_Q = 0$. All parameters in numerical simulations have been explicitly provided, closely consistent with those values reported in previous studies^{32,33,43–46,61}. We have demonstrated that these system parameters satisfy the stability around the blue-sideband resonances using both the Routh-Hurwitz criterion⁷³ and stable limit-cycle solutions⁷⁴. By these two methods on elaborating the dynamical stability, we have demonstrated that all parameter values work in the stable zone (see Supplementary Information). We also highlight the enhanced stability in the nonreciprocal phase and its potential relevance for practical applications in quantum technologies. Specifically, it not only enables one-way quantum manipulation but also contributes to the significant robustness of purely quantum effects against thermal noise and random fabrication imperfections of practical devices. These findings may be advantageous for the implementation of purely quantum behaviors in noisy or engineered quantum environments.

Although what we have proposed a purely theoretical scheme, our approach is completely experimentally feasible, using the state-of-the-art experimental conditions (see Supplementary Information). It demonstrates the consistency between the parameters used in our numerical simulations^{33,43,46} and those reported in realistic experiments^{32,44,45,61}, indicating that the proposed phenomena are experimentally accessible with current state-of-the-art platforms. While the values of quantum synchronization parameters largely follow those in Ref.⁴³, they remain highly effective across a broad range of parameter regimes (see Supplementary Information).

We emphasize the methodological soundness and the high standards maintained throughout this work. Our theoretical framework is based on a set of quantum Langevin equations that capture the coupled dynamics of optical, mechanical, and magnonic modes in the presence of both intrinsic dissipation and external driving. This approach is well established in the studies of cavity optomechanics and optomagnomechanics, and has been carefully adapted to combine the key features of our hybrid platform, including the magnon-Kerr nonlinearity⁴⁴ and the optical Sagnac effect³². We have verified the validity of our methodology by performing extensive numerical simulations across a wide range of state-of-the-art realistic experimental parameters, ensuring that our main effects are not artifacts of fine-tuning (see Supplementary Information). Furthermore, the quantum synchronization dynamics are quantified using a continuous-variable measure consistent with the prior well-known literature⁴³, and the emergence of purely quantum nonreciprocity is traced analytically to the combination of the Sagnac effect³² and magnon-Kerr nonlinearity⁴⁴. The methodology not only meets but also extends current standards in the field of quantum synchronization, by providing a unified framework to study hybrid, nonlinear, and unidirectional quantum phenomena in a tunable optomagnomechanical system.

Nonreciprocal quantum synchronization

One-way quantum synchronization can be induced on demand via the synergy of the Sagnac and magnon-Kerr effects. In the absence (presence) of the Sagnac effect, S_Q is independent (dependent) of the driving direction, under $K = 0$. Specifically, for a spinning device (i.e., $\Omega \neq 0$), when quantum synchronization is generated by injecting light from the right of the fibre, no quantum synchronization happens by applying it from the left, giving rise to a unique nonreciprocity of quantum synchronization, as shown in Fig. 1(d). Physically, the phonon

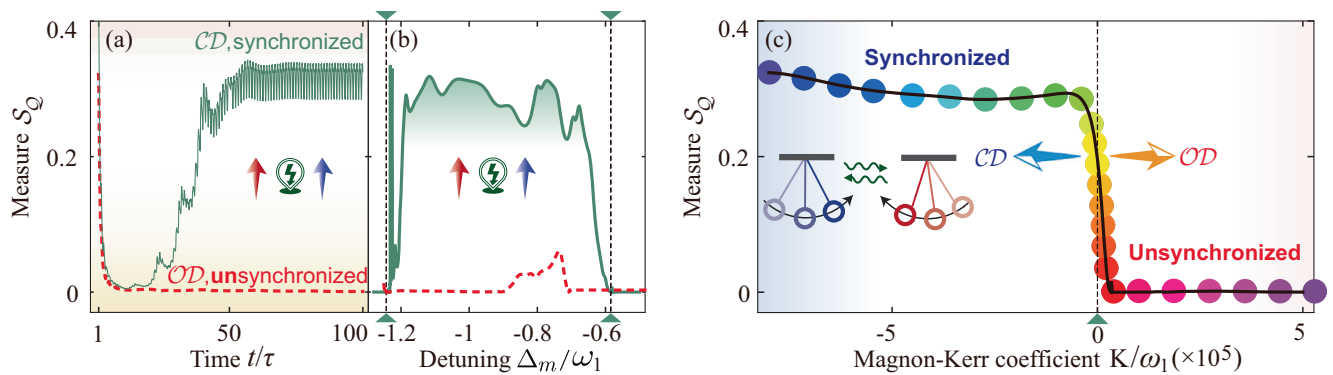


Fig. 2 | Magnon-Kerr-induced nonreciprocal quantum synchronization. **a** Quantum synchronization measure S_Q versus time t , scaled by $\tau = 2\pi/\omega_1$, when the magnetic field enters from the CD ($K/\omega_1 = -5 \times 10^{-5}$) or OD ($K/\omega_1 = 5 \times 10^{-5}$). **b** S_Q

versus the detuning Δ_m for the CD and OD cases. **c** S_Q versus the magnon-Kerr strength K . Here we set $\Omega = 0$ and other parameters are the same as those in Fig. 1.

mode scatters the input laser into anti-Stokes and Stokes sidebands, and optomechanical correlations occur once one of the sidebands is resonant with the optical mode. When the Sagnac effect is turned off, an optomechanical-induced blueshift of the optical mode results in a spectral offset. However, by employing the Sagnac effect, an opposite Sagnac-Fizeau shift occurs and modifies the resonance conditions of the countercirculating light modes, leading to a fundamentally different quantum synchronization once reversing the input-laser direction.

For a non-spinning device (i.e., $\Omega = 0$), an external magnetic field aligned parallel to the CD of the crystallographic axes of the YIG sphere yields a shift in the magnon frequency; whereas it applied along the OD leads to an opposite shift. As a result, quantum synchronization appears for the CD; while for the OD, no quantum synchronization occurs, which demonstrates the emergence of nonreciprocal quantum synchronization, as shown in Fig. 2(a). In addition, we see from Fig. 2(b) that around $\Delta_m/\omega_1 = -1$, nonreciprocal quantum synchronization happens, because it is generated in the CD but vanished in the OD, which fully agrees with the results in Fig. 2(c). The sharp switch between synchronization and unsynchronization is caused by an unparalleled magnon-Kerr-nonlinearity induced transition, which resembles a kind of dynamical phase transition, where the measure of quantum synchronization plays the role of an order parameter. Physically, an extraordinary transition between redshift and blueshift results in the resonance and off-resonance between the detuning and phonons, inducing a completely different physical process.

Robust quantum synchronization

Large mass and high dissipation are frequently manifestations of random fabrication imperfections in quantum devices. Deviations in etching depth, layer uniformity, or material composition can lead to increased inertial mass; while microscopic defects, impurities, and surface roughness introduce unwanted dissipation. Such imperfections can significantly impair quantum coherence and quantum control, highlighting the imperative for ultrahigh-precision shielding in scalable quantum technologies.

Fragile quantum synchronization is easily destroyed by random fabrication imperfections of practical devices. Nevertheless, our approach paves a feasible strategy to shielding the fragile quantum synchronization from the device detriments, enabling the construction of an imperfection-tolerant quantum synchronization. We reveal in Fig. 3a, b that in the magnon-Kerr-off regime, phonons are unsynchronized with increasing mass or decay; in stark contrast to this, they become synchronized in the magnon-Kerr-on regime. The reduction in quantum synchronization results from decreasing coupling strengths with increasing the microsphere-resonator mass; while the

diminishment can be considerably compensated via the magnon-Kerr effect improving both the magnon and photon numbers.

The generating quantum synchronization provides a new way enabling practical noise-sensitive setups to be ideal, which is beneficial for achieving noise-tolerant quantum resources. We show in Figs. 3c, d that in the magnon-Kerr-off regime, quantum synchronization is generally deteriorated or even completely destroyed with increasing thermal phonon number $\bar{n}_j$; while in the magnon-Kerr-on regime, it can be dramatically improved, which approaches to or even surpasses that using an ideal quantum device. This means that the noise-caused detrimental reflection is significantly suppressed with such a magnon-Kerr-nonlinearity device; as a result, a nearly ideal quantum synchronization can be achieved even in a highly imperfect setup. This noise tolerance can also be clearly shown by defining a quantum-synchronization-revival factor:

$$\Lambda = \frac{\max[S_Q(\bar{n}_j \neq 0, K \neq 0)]}{\max[S_Q(\bar{n}_j = 0, K = 0)]}. \quad (7)$$

Surprisingly, the resulting quantum synchronization survives in a noise environment by introducing the magnon-Kerr effect, as shown in Fig. 3(e). Physically, injecting the magnetic field from the CD port yields a nearly symmetric coupling ($G_m \approx G_a$), which enhances the system's resilience against thermal fluctuations and fabrication-induced imperfections, as shown in Fig. 3(f). In contrast, injection from the OD port induces an asymmetric coupling ($G_m \neq G_a$), thereby compromising the robustness of the resonator resilience [see Fig. 3(f)].

Noteworthiness, significance, and advantages

We here highlight three noteworthy and novel results of our work. (i) Nonreciprocal quantum synchronization of phonons remains unexplored. To our knowledge, we are the first to study nonreciprocal quantum synchronization via the synergy of the Sagnac and magnon-Kerr effects. While the use of the Sagnac effect to achieve the nonreciprocity of the optical transmission³², photon blockade³³, and optomechanical entanglement³⁴ has been studied, its application to quantum synchronization has not been explored to date. Inspired by the Sagnac-effect-induced nonreciprocity³², we introduce a fundamentally different nonreciprocity mechanism using the magnon-Kerr nonlinearity, and demonstrate the first realization of nonreciprocal quantum synchronization. (ii) The proposed idea is not a simple synergy of the Sagnac and magnon-Kerr effects, but rather the generation of novel nonreciprocal quantum phenomena and addressing an outstanding challenge, i.e., quantum synchronization is extremely sensitive to random fabrication imperfections and thermal noise of practical devices. Specifically, quantum synchronization is generally deteriorated or even completely destroyed by thermal noise and

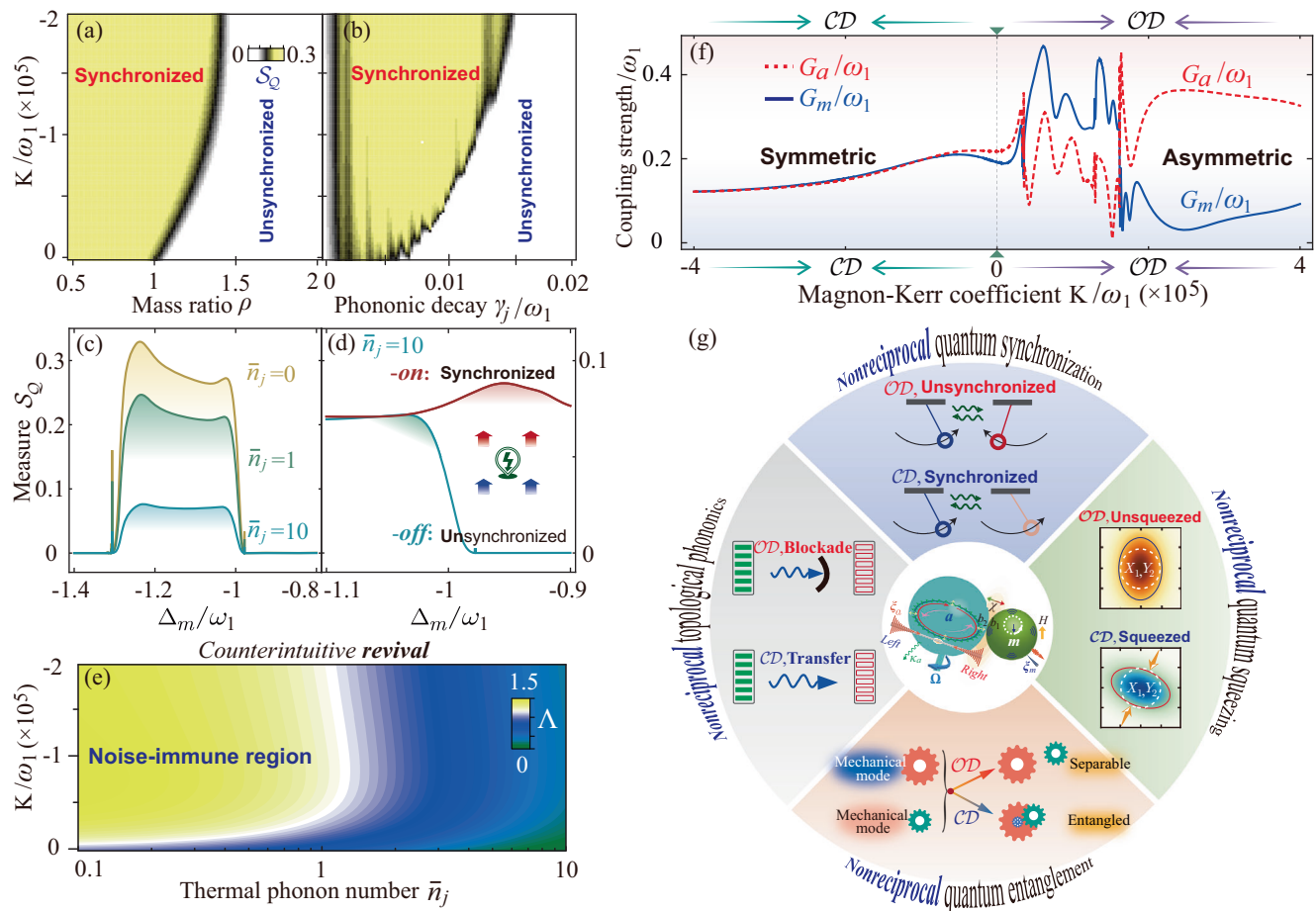


Fig. 3 | Imperfection-tolerant quantum synchronization. **a** S_Q versus the mass ratio $\rho = m_j/m_0$ and K . **b** S_Q versus the phonon decay rate γ_j/ω_1 and K . Although the masses (decay rates) of the two microspheres generally differ, for simplicity, we assume equal masses (decay rates) in our simulations. **c** In the magnon-Kerr-off regime ($K = 0$), S_Q versus Δ_m when thermal phonon numbers $\bar{n}_j = 0, 1$ and 10 . **d** For $\bar{n}_j = 10$, S_Q versus Δ_m in both magnon-Kerr-off and magnon-Kerr-on ($K/\omega_1 = -2 \times 10^{-5}$) regimes. **e** Density plot of the quantum-synchronization-revival factor Λ versus $\bar{n}_j$ and K . **f** Symmetric and asymmetric couplings. Effective coupling $G_{m(a)}$ (see Supplementary Information) of the magnon (photon) and first (second) phonon mode versus the magnon-Kerr strength K . The magnetic field entering

from the CD results in symmetric coupling, corresponding to enhancing the resilience against both thermal fluctuations and random fabrication imperfections of the resonator; while the magnetic field injected from the OD leading to asymmetric coupling, corresponding to reducing the resonator resilience. **g** Broad applicability and universality of our model. The proposed framework naturally extends to a range of nonreciprocal quantum phenomena, including nonreciprocal quantum synchronization, nonreciprocal quantum squeezing, nonreciprocal quantum entanglement, and nonreciprocal topological phononics. Here we set $\Omega = 0$ and other parameters are the same as those in Fig. 1.

random fabrication imperfections. Surprisingly, our proposal overcomes this obstacle and generates a unique one-way quantum synchronization robust to these detrimental factors, without the need of using any high-cost, low-loss materials or noise filters at the expense of the system complexity. In a broader view, our study presents an innovative approach to reversing the intrinsically detrimental effects of practical devices. It paves a general route to pioneering nonreciprocal quantum resources, with counterintuitive robustness against both random fabrication imperfections and thermal noise of practical devices. (iii) The proposed approach has a broad applicability and universality. Our framework is not limited to a specific unidirectional quantum effect such as nonreciprocal quantum synchronization, but naturally extends to a broader class of one-way quantum phenomena, including nonreciprocal quantum entanglement, one-way quantum mechanical squeezing, and unidirectional topological phonon transfer, as shown in Fig. 3(g).

We next emphasize the significance of our findings to the broader scientific community. We believe this work offers both conceptual and technical advances that are broadly relevant to the field of cavity optomechanics. The demonstrated ability to achieve nonreciprocal quantum synchronization via both the Sagnac effect and

magnon-Kerr nonlinearity opens a new pathway for active control of one-way nonequilibrium quantum dynamics in hybrid quantum platforms. These results are expected to be of interest also to researchers in quantum phononics, nonlinear quantum dynamics, and quantum information, where robust and tunable quantum synchronization and its quantum nonreciprocity are highly desirable.

Finally, we provide a detailed comparison with previously established demonstrations to highlight the originality and advantages of our approach. Prior studies have investigated quantum synchronization in optomechanical systems⁴³, nonreciprocal transport of information or photons using the Sagnac effect^{32,33}, and magnon-Kerr nonlinearity in YIG-based systems^{44,47,49}. However, our work brings together these ingredients in a previously unexplored regime: (i) The sign and strength of the magnon-Kerr nonlinearity are tuned in situ via the external magnetic field, allowing dynamical control over the coupling landscape. (ii) Nonreciprocal quantum synchronization is modulated by optical driving directions or magnetic field directions, enabled by the Sagnac or magnon-Kerr effects, respectively. (iii) Quantum synchronization becomes effectively nonreciprocal, without requiring additional gain or engineered reservoirs. To our knowledge, no previous study has demonstrated this level of nonreciprocal

control of quantum synchronization via combined photonic, phononic, and magnonic pathways. We are neither aware of any prior established demonstration that reports this combination of these mechanisms, nor one that realizes tunable nonreciprocal quantum synchronization in this manner.

Feasible experimental implementations

In a realistic experiment⁶¹, the hybrid quantum platform comprises a YIG sphere (as a magnomechanical cavity) and a silica microsphere (as an optomechanical cavity), both of which are coherently coupled to each other via direct physical contact. In the silica microsphere, the phonon mode is driven by the radiation-pressure interaction from circulating optical fields; whereas in the YIG sphere, it is excited via magnetostrictive forces mediated by microwave-driven magnons.

Specifically, a uniform magnon mode, supported by the YIG sphere under an external magnetic field, couples to a phonon mode through magnetostrictive interaction^{44–49}, enabling microwave excitation of phonons in the YIG sphere. In the silica microsphere supporting a radiation-pressure-induced mechanical radial breathing mode, the optical mode and the mechanical radial mode are intrinsically coupled through radiation pressure and the photoelastic effect^{64,79–81}, forming a canonical optomechanical interaction. Meanwhile, direct physical contact between the silica and YIG microspheres establishes an effective mechanical coupling of their localized phonon modes. In the YIG sphere, microwave driving of the magnon mode excites the phonon mode via the magnomechanical effect⁸². Bringing the silica microsphere into direct contact with the YIG sphere establishes a mechanical coupling between their spatially separated phonon modes. Simultaneously, the radiation-pressure-induced optomechanical coupling in the silica cavity plays a key role in the hybrid quantum dynamics. The process involves a synergistic interplay of optomechanics, magnomechanics, phonon interference, and quantum mechanical effects^{83,84}, wherein microwave and optical signals are coherently mapped onto two nearly degenerate mechanical modes, enabling their interference^{85,86}.

Discussion

Here we delineate in detail the fundamental differences between nonreciprocal quantum synchronization and nonreciprocal quantum steering⁸⁷. Nonreciprocal quantum synchronization and nonreciprocal quantum steering both exhibit unidirectional quantum behavior in quantum systems, but arise from fundamentally different physical mechanisms. Nonreciprocal quantum synchronization refers to asymmetric quantum dynamical locking, such as phase or frequency entrainment, between coupled quantum oscillators. This unidirectionality stems from engineered asymmetries in the considered quantum system via the Kerr nonlinearity or rotation-induced Sagnac effect, leading to one-way quantum coherence in time-domain observables. The resulting unidirectional quantum coherence emerges in the time evolution of system observables and reveals asymmetric (one-way) quantum synchronization. In stark contrast, nonreciprocal quantum steering is a form of asymmetric quantum correlation and a measurement-based manifestation of quantum nonlocality⁸⁷, wherein one party (Alice) can nonlocally affect quantum state of another's part (Bob) through measurement, but not vice versa. This irreversibility reflects a directional violation of local hidden state models and underpins one-sided device-independent quantum protocols. While both phenomena break reciprocity, quantum synchronization concerns quantum dynamical behavior, whereas quantum steering reflects the structure of quantum measurement correlations. That means that, unlike quantum synchronization, quantum steering does not arise from quantum dynamical evolution but from the structure of quantum measurements and conditional states.

In addition, we show some discussions on the potential applications and advantages of our findings. The demonstrated

nonreciprocal control of quantum synchronization in this hybrid platform may find use in unidirectional quantum information processing, where synchronized quantum systems can serve as robust building blocks for distributed quantum networks^{78,87–90}. The tunable magnon-Kerr nonlinearity effect and its resilience to random fabrication imperfections of practical devices suggest possible applications in quantum sensing and quantum signal transduction, especially in noisy or imperfect environments. Our robustness analysis provides insights relevant for the design of scalable chiral quantum networks, where fabrication-induced imperfections are inevitable. These findings may also contribute to future developments in nonreciprocal quantum sensing architectures that exploit quantum collective dynamics for enhanced quantum precision^{78,87–90}. In particular, the proposed framework on nonreciprocal quantum synchronization unlocks multiple exciting opportunities for application across quantum technologies. Including: (i) Nonreciprocal quantum information processing. The resulting nonreciprocal quantum synchronization enables a controllable unidirectional flow of quantum correlations (quantum information), which can be harnessed for unidirectional quantum signal routing in phononic or hybrid quantum networks, where thermal robustness and coherence preservation are essential⁷⁸. (ii) Nonreciprocal quantum state engineering. Our scheme offers a tunable, nonlinearity-engineered route to stabilize phase-locked mechanical states. This can be employed to prepare non-classical mechanical states, which is good for nonreciprocal quantum sensing or interface protocols between mechanical and optical (magnonic) degrees of freedom^{87,88}. (iii) Quantum transduction architectures. In hybrid quantum systems where mechanical resonators serve as intermediaries between disparate platforms (e.g., microwave-to-optical conversion), nonreciprocal quantum synchronization could enable efficient and noise-robust temporal alignment across subsystems^{89,90}. (iv) Fundamental studies of irreversibility. The intrinsic unidirectionality in the quantum synchronization dynamics constitutes a controlled setting for investigating microscopic origins of irreversibility and entropy production in open quantum systems, thus offering insights relevant to nonreciprocal quantum thermodynamics. Please see Supplementary Figs. S1–S11 and Supplementary Tables I–IV for more details.

In conclusion, we showed nonreciprocal quantum synchronization of phonons arising from the Sagnac and magnon-Kerr effects, without which it vanishes. Remarkably, it is robust to random fabrication imperfections and thermal noise of practical devices. Our work maps a new perspective on constructing a fundamentally different nonreciprocal quantum device, with the robustness against imperfections and noise.

Data availability

The authors declare that the data supporting the findings of this study are available within the paper and its supplementary information files.

Code availability

The code used for the current study is available from the corresponding author on reasonable request.

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Author contributions

D.G.L. conceived the project and performed calculations. D.G.L., A.M., and F.N. interpreted the results. F.N. supervised the project. All authors contributed in writing the manuscript.

Competing interests

The authors declare no competing interests.

Additional information

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