

MODIFICATION OF SQUEEZING IN THE KICKED ANHARMONIC OSCILLATOR¹

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Abstract

We discuss a model of a cavity filled with a passive nonlinear ‘Kerr’ medium and periodically kicked by a series of ultra-short laser pulses. We perform numerical calculations to find the variance of the field quadrature to determine the squeezing properties of the cavity field. We show that degree of the squeezing depends on the parameters describing external, pulsed excitation.

1 The model

One of the most commonly discussed models of quantum optics is that with a cavity filled with a nonlinear oscillator. It is known that the systems with anharmonic oscillator can exhibit squeezing properties [1,2]. In this paper we shall concentrate on the system involving anharmonic oscillator modelled by a nonlinear medium that is irradiated by series of ultra-short coherent pulses. Our system can be realized, for instance, by a cavity filled with nonlinear Kerr medium. This cavity is irradiated by series of ultra-short coherent pulses.

The system discussed here is governed by the following interaction Hamiltonian (in units of $\hbar = 1$):

$$\hat{H}_{int} = \frac{\chi}{2} (\hat{a}^\dagger)^2 \hat{a}^2 + (\epsilon \hat{a}^\dagger + \epsilon^* \hat{a}) f(t) , \quad (1)$$

where the envelope-function $f(t)$ models the series of ultra-short pulses in a form of Dirac-delta functions as follows:

$$f(t) = \sum_{n=0}^{\infty} \delta(t - nT) . \quad (2)$$

The complex amplitude ϵ appearing in (1) determines the strength of the pulses and is related to the external – cavity fields interaction, whereas the nonlinearity parameter χ describes the nonlinear interaction of the cavity field with itself which is mediated by the nonlinear medium (self-phase modulation).

To investigate the time-evolution of our system we shall perform numerical calculations based on the method applied in [3-6]. Owing to the fact that the ultra-short pulses are modeled by the Dirac delta functions the time-evolution of the system can be divided into two stages of different nature. The first stage is a ‘free’ evolution during the time between two subsequent pulses and

¹published in: Proceedings of the Fifth Int. Conf. on *Squeezed States and Uncertainty Relations* (Balatonfured, Hungary, 1997), eds. D. Han *et al.*, NASA Conf. Publication No. 206855 (NASA, Greenbelt, 1998) 233-238.

is determined by the first term of the Hamiltonian (1) ($\frac{\chi}{2} (\hat{a}^\dagger)^2 \hat{a}^2$). This evolution is described by the following unitary operator

$$\hat{U}_0 = \exp(-i\frac{\chi T}{2} (\hat{a}^\dagger)^2 \hat{a}^2) , \quad (3)$$

where the time T determines the interval between two subsequent pulses. The second stage of the time-evolution of the system is caused by its interaction with the infinitely short pulse. This part of the evolution is described by the second term (proportional to ϵ) of the Hamiltonian \hat{H}_{int} , given by (1). Thus, the evolution operator corresponding to the interaction during a single pulse can be written as

$$\hat{U}_1 = \exp(-i\epsilon(\hat{a}^\dagger + \hat{a})) . \quad (4)$$

In consequence the overall evolution of the state of the system can thus be described as a subsequent action of the operators \hat{U}_0 and \hat{U}_1 on the initial state. Assuming that for the time $t = 0$ the system was in the state $|\Phi_0\rangle$ we express the state $|\Phi_k\rangle$ just after k -th kick as

$$|\Phi_k\rangle = (\hat{U}_1 \hat{U}_0)^k |\Phi_0\rangle . \quad (5)$$

With this procedure we are able to obtain the state after an arbitrary, k -th pulse, and hence, the mean value of the moments involving annihilation and creation operators describing the field.

Since we are interested in the squeezing properties of our system we introduce, similarly as in [1], the following quadrature operators:

$$\hat{Q}_\phi = \hat{a} \exp(-i\phi) + \hat{a}^\dagger \exp(i\phi) . \quad (6)$$

As the phase ϕ is equal to zero the parameter Q is the in-phase quadrature, whereas for $\phi = \pi/2$ the quantity Q is the out-of-phase quadrature. To investigate the squeezing properties of the system we shall introduce normally ordered variance

$$V_\phi = \langle : \hat{Q}_\phi^2 : \rangle - \langle \hat{Q}_\phi \rangle^2 . \quad (7)$$

This quantity can be expressed by the following formula [2]:

$$V_\phi = 2Re \left\{ \langle \hat{a}^2 \rangle e^{-2i\phi} - \langle \hat{a} \rangle^2 e^{-2i\phi} \right\} + 2 \left\{ \langle \hat{a}^\dagger \hat{a} \rangle - \langle \hat{a}^\dagger \rangle \langle \hat{a} \rangle \right\} \quad (8)$$

The sign of the variance V_ϕ is related to the squeezing properties of the field – for $V_\phi < 0$ the state is squeezed, and as the value of V_ϕ reaches -1 the state is perfectly squeezed.

2 Numerical results

Since this paper is devoted to the squeezing properties of the field generated by our model we perform numerical calculations enabling us to find the time-evolution of the variances V_ϕ of the quadrature operators Q . Moreover, we compare our results with those of Tanaš [2] obtained analytically for the nonlinear oscillator. However, one should keep in mind that our system differs from that discussed in [2] in fact that we have involved series of ultra-short pulses driving nonlinear media.

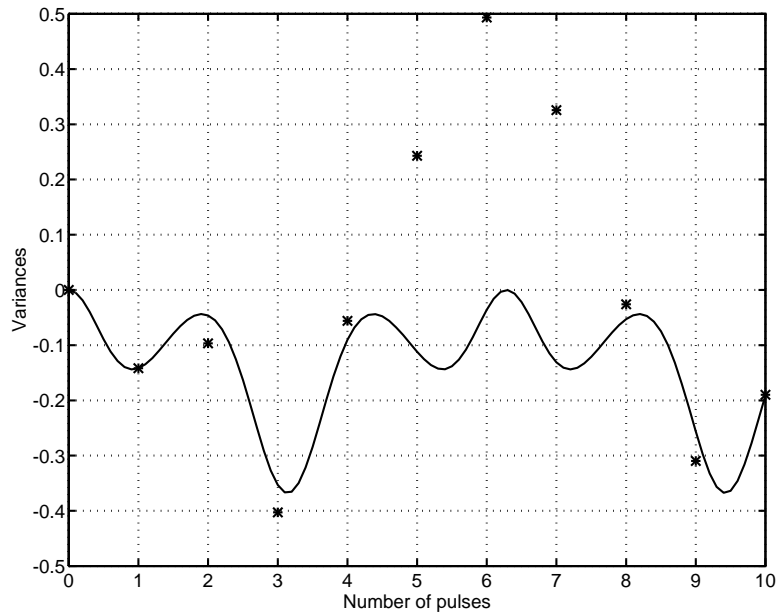


FIG. 1. The variances V_ϕ for the unperturbed (solid line) and kicked nonlinear Kerr media (star marks). The parameters $|\epsilon| = 0.05$, $T = 1$ and $\phi_\epsilon = -\pi/2$. All energies are measured in units of $\chi = 1$.

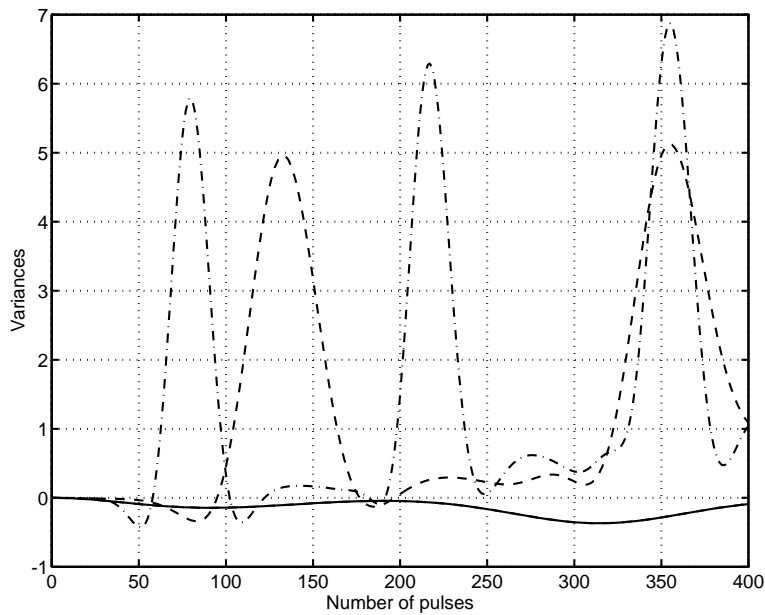


FIG. 2. The same as Fig.1 but for various values of the time T : dash-dotted line – $T = 0.01$, dashed line $T = 0.02$, solid line – unperturbed system.

Thus, Fig.1 shows the case when the field was initially in the coherent state with the mean number of photons equal to 0.25. The strength of the pulses $|\epsilon| = 0.05$, their phase $\phi_\epsilon = -\pi/2$ and the time T between two subsequent pulses is equal to 1. We use the notation $\epsilon = |\epsilon| \exp(i\phi_\epsilon)$ and assume that all energies are measured in units of χ . To compare our results with those of [2] we plot the in-phase variance component for the unperturbed system (solid line), whereas, the star marks show this component just after each subsequent pulse. We see that the amplitudes of oscillations of the variance increases due to the presence of the external excitation. In consequence, the minima become deeper and, hence, the squeezing can increase.

Fig.2 corresponds to the same situation as that shown in Fig.1 but the time T is assumed to be much smaller then for the previous case: $T = 0.01$ (dash-dotted line) and $T = 0.02$ (dotted line). Solid line corresponds to the unperturbed system again. It is seen that the variance starts to oscillate very rapidly as we compare it with the unperturbed system. In consequence, new minima appear and the squeezed state can be achieved for shorter times than for the system discussed in [2]. Moreover, it is seen that the amplitudes and frequencies of the oscillations depend on the value of time T .

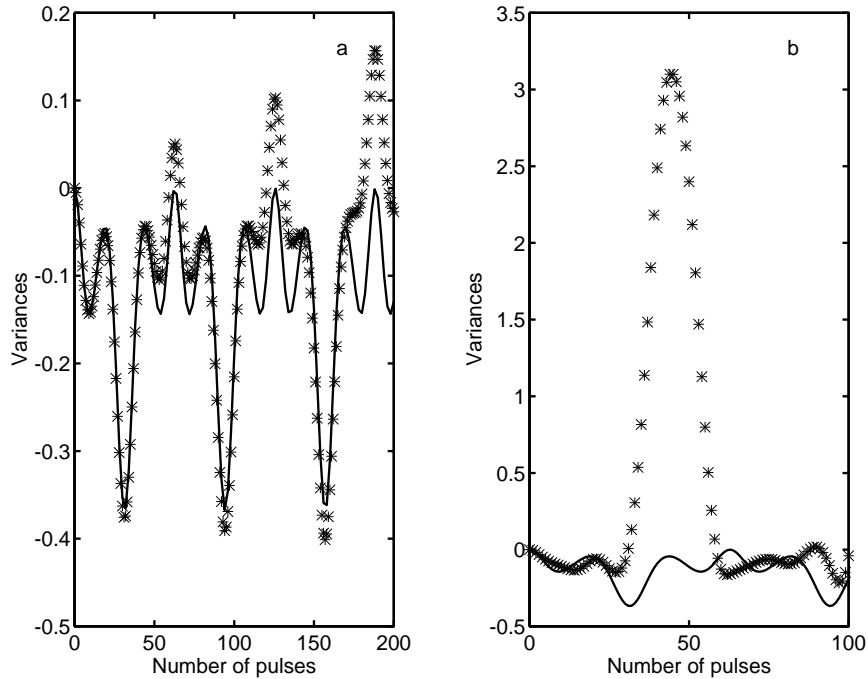


FIG. 3. The variances for the same parameters as in Fig.1 but for various values of the coupling constant ϵ — $|\epsilon| = 0.001$ (Fig.3a) and $|\epsilon| = 0.05$ (Fig.3b). Solid lines correspond to the unperturbed system.

The dependence of the variance V_ϕ on the strength of the external coupling is shown in Figs. 3a and 3b. We assume here that the parameters are the same as for Fig.1 but $|\epsilon| = 0.001$ (Fig.3a) and $|\epsilon| = 0.05$ (Fig.3b). For tiny external excitations (Fig.3a, star marks) the variance oscillates similarly as for the unperturbed system (solid line). The difference occurs in amplitudes of those oscillations. The divergence between those two models becomes more and more pronounced as

the number of pulses increases. As $|\epsilon|$ increases the behavior of the variance changes considerably and is completely different from that for the unperturbed system (Fig.3b). It is seen that our system is very sensitive on the strength of the external excitation.

The last plots show the variance V_ϕ evolution for various values of the phase ϕ_ϵ . Fig. 4 corresponds to the same parameters as those for Fig.1 but the phase $\phi_\epsilon = 0$ (star marks) and $\phi_\epsilon = \pi/2$ (circle marks). It is seen that for the case of $\phi_\epsilon = 0$ squeezing decreases as we compare it with case of the unperturbed system except for the time just after the first pulse. As we assume that $\phi_\epsilon = \pi/2$ the variance is positive for the times after the first pulse – the squeezing is destroyed. We can explain this phenomenon as a result of the dependence of the mean number of photons in the system on the phase of the excitation. The energy can be pumped in or exhausted from the system. In consequence, the squeezing that is relative to the number of photons can increase or decrease accordingly to the phase of the excitations. Moreover, similarly as for the previous figures we parallel our results and those for the system with unperturbed oscillator (solid line).

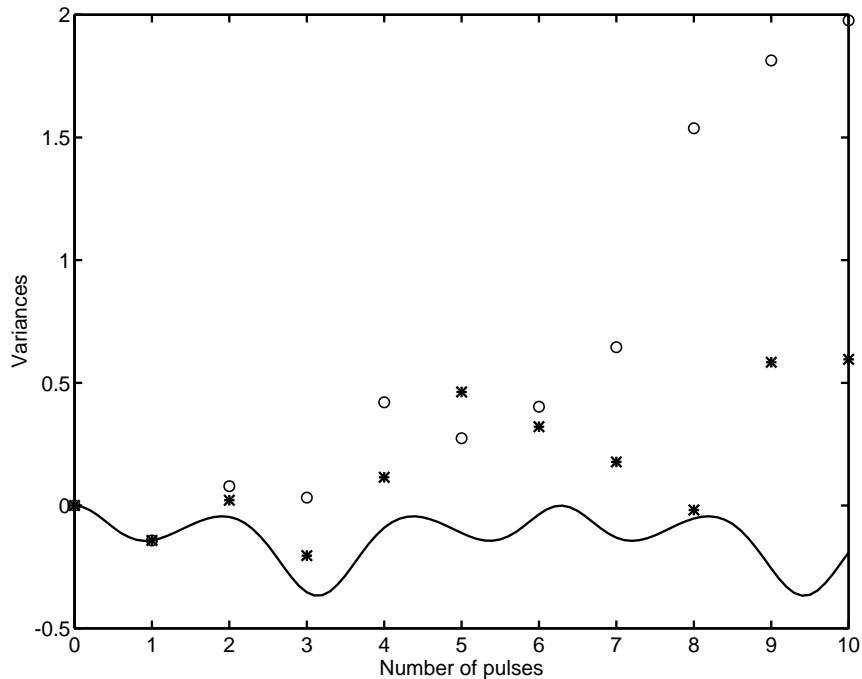


FIG. 4. The variances V_ϕ for various values of the phase ϕ_ϵ : $\phi_\epsilon = 0$ – star marks, $\phi_\epsilon = \pi/2$ – circle marks. The coupling constant $\epsilon = 0.1$. The remaining parameters are the same as in Fig.1. Solid line corresponds to the unperturbed system.

3 Conclusions

The systems comprising Kerr nonlinear media can exhibit strong squeezing properties [1,2]. In this paper we dealt with the same kind of system. However, in addition, we have included in our model external excitations realized by the series of ultra-short coherent pulses. We have shown that this kind of excitation leads to considerably different behavior of the system depending on the

parameters describing external pulses. In consequence, the variance of the quadrature depends not only on the strength of the pulsed external excitation but also on its phase and the time between two subsequent pulses.

Acknowledgments

We acknowledge the Poznań Supercomputing and Networking Center for giving us ability to perform numerical calculations. This work was carried out within the KBN Project 2 P03B 73 13.

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