Non-Hermitian Hamiltonians vs Liouvillians for Atomic Systems

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Non-Hermitian quantum physics has recently garnered significant interest due to its ability to describe energy dissipation in open quantum systems and to predict exotic phenomena such as exceptional points (EPs). In this context, non-Hermitian dynamics is often modeled using non-Hermitian Hamiltonians (NHHs). While these have been successfully applied in various platforms–particularly in photonic systems [1–3]–their use is now extending to a wider range of systems, including thermal atomic ensembles [4]. However, for some systems, an accurate description of the dynamics often requires a more complete treatment beyond NHHs [5–7].

We investigate this discrepancy using a model of an open atomic system with two hyperfine states ($F = 1$ and $F' = 0$), analyzed using the formalism of effective operators [8]. By comparing the eigenvalue spectra obtained from both an NHH and a Liouvillian superoperator, we analyze the emergence and properties of Hamiltonian and Liouvillian exceptional points, as well as diabolical points.

Our results show that, for atomic systems, relying solely on NHHs can be insufficient for accurately capturing the system's spectral features. While NHHs can provide correct predictions in specific scenarios, a full description generally requires the Liouvillian superoperator, which governs the Lindblad master equation and explicitly incorporates quantum jump terms. We demonstrate that the inclusion of quantum jumps via the Liouvillian formalism can fundamentally alter the spectral properties of the system. Specifically, we present examples where the existence, location in parameter space, or the order of spectral degeneracies differ significantly between the two approaches, underscoring situations where the NHH description breaks down.

Furthermore, by leveraging the hybrid-Liouvillian formalism [9], we illustrate how quantum jumps influence the spectral features predicted by the NHH, ultimately shaping the full spectrum as governed by the Liouvillian.

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