Unveiling Singularities in Quantum Bosonic Systems: From Diabolical to Exceptional

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We present a comprehensive study of quantum exceptional points, quantum diabolical points, and quantum hybrid points arising in multimode bosonic systems described by quadratic Hamiltonians. This study, based on a two-part investigation [1-2], focuses on the classification, physical realization, and dynamical implications of these spectral singularities within the framework of open quantum systems. We demonstrate how these singularities manifest directly in the Heisenberg picture dynamics of bosonic operators. We begin with multi-mode bosonic systems composed of up to five coupled modes described by non-Hermitian Hamiltonians under generalized PT-symmetric conditions. These models are physically well-grounded, and their predictions are widely accepted within the field. Thereafter, we address more unconventional configurations, where PT-symmetric behavior is not globally defined but appears only within restricted regions of the Liouville space. We refer to this as nonconventional PT symmetry, and explore its implications for both spectral structure and physical interpretation. We distinguish between inherited and genuine spectral singularities: the former being artifacts of system decomposition, and the latter intrinsic to the reduced system's non-Hermitian dynamics. Finally, we analyze systems with unidirectional coupling, a strongly non-Hermitian interaction that can be realized, for example, via counter-propagating bosonic modes. While such systems enable the engineering of higher-order inherited degeneracies through concatenation, we find that the preservation of quantum consistency (e.g., operator commutation relations and physicality) is limited to short-time dynamics. Our results offer a unified framework for understanding and controlling complex singular behaviors in non-Hermitian bosonic systems and open pathways toward their application in quantum sensing, mode control, and non-Hermitian quantum engineering.

[1] K. Thapliyal, J. Peřina Jr., G. Chimczak, A. Kowalewska-Kudlaszyk, A. Miranowicz, Multiple quantum exceptional, diabolical, and hybrid points in multimode bosonic systems: I. Inherited and genuine singularities, arXiv:2405.01666 (2024).

[2] J. Peřina Jr., K. Thapliyal, G. Chimczak, A. Kowalewska-Kudlaszyk, A. Miranowicz, Multiple quantum exceptional, diabolical, and hybrid points in multimode bosonic systems: II. Nonconventional PT-symmetric dynamics and unidirectional coupling. arXiv:2405.01667 (2024).