Gröbner-based decision trees for quantum negativity detection using moment operators and matrix invariants

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Abstract

Traditional quantum state tomography scales exponentially with system size, making entanglement detection impractical for larger quantum systems. We present a comprehensive method for measuring quantum negativity through moments of a partially transposed density matrix combined with Gröbner basis decision trees that classify quantum states according to their algebraic symmetry properties.

Our approach introduces three types of matrix invariants—standard (I_k), realignment (R_k), and mixed (M_k)—that capture complementary aspects of quantum correlations. By establishing relationships between these invariants and moments of the partial transpose ($\mu_n = I_k + cR_k + dM_k$), we derive Gröbner basis relations that enable construction of optimal decision trees for negativity computation. These trees systematically classify states by their algebraic properties, providing direct computation formulas for common symmetry classes without solving full characteristic polynomials.

We develop quantum circuits for direct measurement of individual invariants using controlled-SWAP (CSWAP) gates, requiring only single-qubit measurements. The moment operators are decomposed into CSWAP sequences, enabling practical implementation on current quantum hardware with polynomial scaling. The method successfully extends to hybrid qubit-qutrit systems through Bell-state encoding, demonstrating versatility across different Hilbert space dimensions.

This framework advances the fundamental understanding of quantum state symmetries and practical entanglement detection in quantum devices.

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